UNIVERSITY OF PANNONIA

DOCTORAL THESIS

Risk-Based Statistical Process Control

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Risk-Based Statistical Process Control

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Publications

Most of the introduced methodologies and figures are previously appeared in the scientific articles listed below:

Thesis 1 (Sections 3.1, 4.1, 5.1):

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Kosztyán, Z. T., & Katona, A. I. (2016). Risk-based multivariate control chart. In: *Expert Systems with Applications*, 62, 250-262. DOI: doi.org/10.1016/j.eswa .2016.06.019

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Abstract

Doctoral School in Management Sciences and Business Administration
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Doctor of Philosophy

Risk-Based Statistical Process Control

by Attila Imre KATONA

Control charts are powerful tools of statistical process control. In the scientific literature, there is a large scale of control charts that can be used under different conditions (e.g., non-normality, autocorrelation etc...) however, most of them disregard the distortion effect of measurement errors. Importance of measurement uncertainty was strongly emphasized by several scholars and in comparison to that, the number of papers dealing with control chart under the presence of measurement error is way below the expectations. Furthermore, these few studies analyzed the effect of the measurement uncertainty but give no detailed and comprehensive solution or propose new control chart that is able to reduce the risk of incorrect decisions. On the other hand, measurement errors are characterized based on the expected value and standard deviation of the distribution function but effect of skewness and kurtosis on conformity / process control performance were not investigated.

In this dissertation, the author provides systematic literature review in order to explore the relevant studies and highlight the deficiencies of control chart design research field. Effect of 3^{rd} and 4^{th} moments (skewness, kurtosis) of measurement error distribution on total inspection and acceptance sampling is analyzed through simulations and several sensitivity analysis are provided. Applying the results of the aforementioned analysis, a new risk-based multivariate (RBT^2) and adaptive (RB VSSI $\overline{X}$) control chart design approaches are proposed with the consideration of measurement uncertainty. Simulations and sensitivity analyses were provided in order to demonstrate the performance of the proposed RBT^2 and RB VSSI X chart under different conditions.

The developed risk-based control charts are able to decrease the amount of type II. errors (prestige loss) by the optimal adjustment of control lines taking measurement uncertainty into account. Process shifts can be detected more precisely in multivariate (RBT^2) or adaptive (RB VSSI $\overline{X}$) cases as well. In addition, even sampling procedure can be rationalized with the RB VSSI $\overline{X}$ chart.

As limitation of the method, the process performance value were estimated where it is still beneficial to consider the effect of measurement errors.

Finally, real practical examples were provided and laboratory experiments were organized to validate the existence of skewed measurement error distribution and verify applicability of the proposed methodology at a company from automotive industry.

UNIVERSITY OF PANNONIA

Zusammenfassung

Doctoral School in Management Sciences and Business Administration
Department of Quantitative Methods

Doktor der Philosophie

Risikobasierte Statistische Prozesskontrolle

von Attila Imre KATONA

Regelkarten sind leistungsfähiges Mitteln von statistische Prozessregelung. Im Fachliteratur, vielfältige Regelkarten wurden entwickelt die unter anderen Rahmenbedingungen (z.B. Nicht-Normalverteilung, Autokorrelation, usw...) verwendbar sind, aber diese Regelkarten berücksichtigen die Effekt von Mesffehler nicht.

Viele Forscher betonten das Wichtigkeit der Messunsicherheit, trotzdem gibt es nur wenige Studien die analisieren die Effekt des Mesffehlers an der Performance den Regelkarten. Zusätzlich diesen wenige Studien fokussieren sich auf die Performance den Regelkarten, aber geben keine Vorschlag zur Behandlung von Messunsicherheit oder zur Reduktion den Entscheidungsrisiken. Messfehlers werden andererseits gekennzeichnet durch Erwartungswert und Standardabweichung des Distributions aber Auswirkungen von Schiefe und Kurtosis auf Prozesskontrolle wurden nicht analisiert.

In dieser Dissertation, der Autor führtet eninen systematischen Literatur Durchsicht um relevanten Artikeln zu erkunden und Mängel im Literatur zu markieren. Auswirkungen von Schiefe und Kurtosis des Messunsicherheitdistributions auf Konformitätskontrollstrategie wurden durch Simulationen und Sensitivitätsanalysen untersucht.

Ergebnisse den Simulationen wurden verwenden um neuen risikobasierte multivariate (RBT^2) und adaptive Regelkarten (RB VSSI $\overline{X}$) zu entwickeln. Beide vorschlägten Regelkarten konnten die Menge den Typ-II Fehlern reduzieren durch die Optimierung den Kontrollgrenzen. Die Veränderung von Prozess Erwartungswert kann effektiv identifizieren werden und auch Stichprobenverfahren kann rationalisiert werden mit die adaptive riskbasierte Regelkarte.

Als Beschränkung, das Wert dem Prozessleitungsfähigkeitsindex wurde geschätzt, womit die Berücksichtigung von Messunsicherheit noch sinvoll ist.

Praktische Beispiele und laboratorische Experimenten wurden schließlich bereitgestellt um die eingeführten Methoden zu prüfen und die Anwendbarkeit zu demonstrieren.

Contents

List of Figures

List of Tables

List of Abbreviations

ANOVA	**An**alysis **O**f **Va**riance
BIPM	**B**ureau **I**nternational des **P**oids et **M**esures
CL	**C**ontrol **L**imit
GA	**G**enetic **A**lgorithm
GRR	**G**age **R**epeatability and **R**eproducibility
GUM	**G**uide to the Expression of **U**ncertainty in **M**easurement
ILAC	**I**nternational **L**aboratory **A**ccreditation **C**ooperation
JCGM	**J**oint **C**ommittee for **G**uides in **M**etrology
MCS	**M**onte-**C**arlo **S**imulation
MSA	**M**easurement **S**ystem **A**nalysis
NIST	**N**ational **I**nstitute of **S**tandards and **T**echnology
NM	**N**elder-**M**ead direct search algorithm
PRISMA	**P**referred **R**eporting **I**tems for **S**ystematic Reviews and **M**eta-**A**nalyses
SPC	**S**tatistical **P**rocess **C**ontrol
RB	**R**isc-**B**ased (aspect)
RBT^2	**R**isk-**B**ased T^2 control chart
RB VSSI $\overline{X}$	**R**isk-**B**ased $\overline{X}$ control chart
VSI	control chart with **V**ariable **S**ampling **I**nterval
VSS	control chart with **V**ariable **S**ample **S**ize
VSSI	control chart with **V**ariable **S**ample **S**ize and sampling **I**nterval
VP	control chart with **V**ariable **P**arameters
WL	**W**arning **L**imit

List of Symbols

Control charts and product characteristics

a	outcome based on real T^2
b	outcome based on detected T^2
CL	central line
$\mathbf{E}$	measurement error matrix
ε	measurement error
h	sampling interval
k	control limit coefficient
K_{LSL}	correction component of LSL
K_{USL}	correction component of USL
LCL	Lower Control Limit
LSL	Lower Specification Limit
LWL	Lower Warning Limit
μ_ε	expected value measurement error
μ_x	expected value of real product characteristic
μ_y	expected value of detected product characteristic
n	sample size
p	number of controlled product characteristics
P_{pk}	process performance index
σ_ε	standard deviation of measurement error
σ_x	standard deviation of real product characteristic
σ_y	standard deviation of detected product characteristic
T^2	Hotelling's T^2 statistics for real values
$\widehat{T^2}$	Hotelling's T^2 statistics for detected values
UCL	Upper Control Limit
UCL_{RBT^2}	control limit of RBT^2 chart
UCL_{T^2}	control limit of traditional T^2 chart
USL	Upper Specification Limit
UWL	Upper Warning Limit
w	warning limit coefficient
x	real product characteristic
$\mathbf{X}$	matrix of real product characteristic values
$\overline{x}$	real sample mean
y	detected product characteristic
$\mathbf{Y}$	matrix of detected product characteristic values
$\overline{y}$	detected sample mean

Cost components

c_{00}	cost of correct rejection
c_{01}	cost of incorrect acceptance
c_{10}	cost of incorrect rejection
c_{11}	cost of correct acceptance

c_f	cost of false alarm identification
c_i	cost of intervention
c_{id}	cost of delayed intervention
C_{ij}	aggregated cost of decision outcome
c_{ma}	maintenance cost
c_{mf}	fixed cost of measurement
c_{mi}	cost of missed intervention
c_{mp}	proportional cost of measurement
c_p	production cost
c_q	cost of qualification
c_r	cost of restart
c_{rc}	cost of root cause search
c_s	cost of switching
d_1	weight parameter for switching
d_2	weight parameter for intervention
ΔC	cost reduction rate
N_h	produced quantity in the considered interval (h)
q_{ij}	count of each decision outcome
TC	total decision cost

Optimization - RBT^2

α	reflection parameter
β	expansion parameter
γ	contraction parameter
δ	shrinking parameter
K_E	expansion point (regarding K)
K_{IC}	inside contraction point (regarding K)
K_{OC}	outside contraction point (regarding K)
K_R	reflection point (regarding K)

Optimization - RB VSSI $\overline{\mathbf{X}}$

C_B	best vertex
C_G	good vertex
C_W	worst vertex
$\mathbf{v}_R$	vector of reflection point coordinates
C_R	cost function value in reflection point
$\mathbf{v}_E$	vector of expansion point coordinates
C_E	cost function value in expansion point
$\mathbf{v}_{IC}$	vector of inside contraction point coordinates
$\mathbf{v}_{OC}$	vector of outside contraction point coordinates
C_{OC}	cost function value in outside contraction point
C_{IC}	cost function value in inside contraction point
$\mathbf{v}_{2s}$	vector of shrinking point coordinates (n$^t h$ vertex)
$\mathbf{v}_{3s}$	vector of shrinking point coordinates (n+1$^t h$ vertex)

Chapter 1

Introduction

1.1 Motivation of the Thesis

Control charts are powerful tools of production management. In case of process shift, the chart gives signal and the production equipment can be maintained in order to avoid the increased number of defective products.(Montgomery, 2012, Woodall and Montgomery, 1999, Kemény et al., 1998). Furthermore, the process is "in-control" when the value of the product characteristic falls within the statistically determined control limits (Shewhart, 1931, Besterfield, 1994).

The traditional control chart philosophy does not consider the risks of the decisions, however, every decision in the process control is distorted by different sources like sampling or measurement uncertainty (Hegedűs et al., 2013a, Katona, 2013) This thesis focuses on decision risks caused by the uncertainty of measurement, because measurement errors can be modeled well and the distribution of errors can be easily simulated.

Although consideration of measurement uncertainty is not included in traditional control chart design approach, producers' and suppliers' risks are frequently discussed topics in conformity or process control (Lira, 1999). If the measuring device or the measurement process is not accurate enough, incorrect decisions (e.g., unnecessary stoppage or missed maintenance) can be made. (Pendrill, 2008). Therefore, the rate of producer's and customer's risk is strongly depending on the measurement uncertainty, leading to prestige loss for the manufacturer company. Measurement errors can occur in conformity control and statistical process control as well. Figure 1.1 illustrates the effect of measurement errors on conformity assessment.

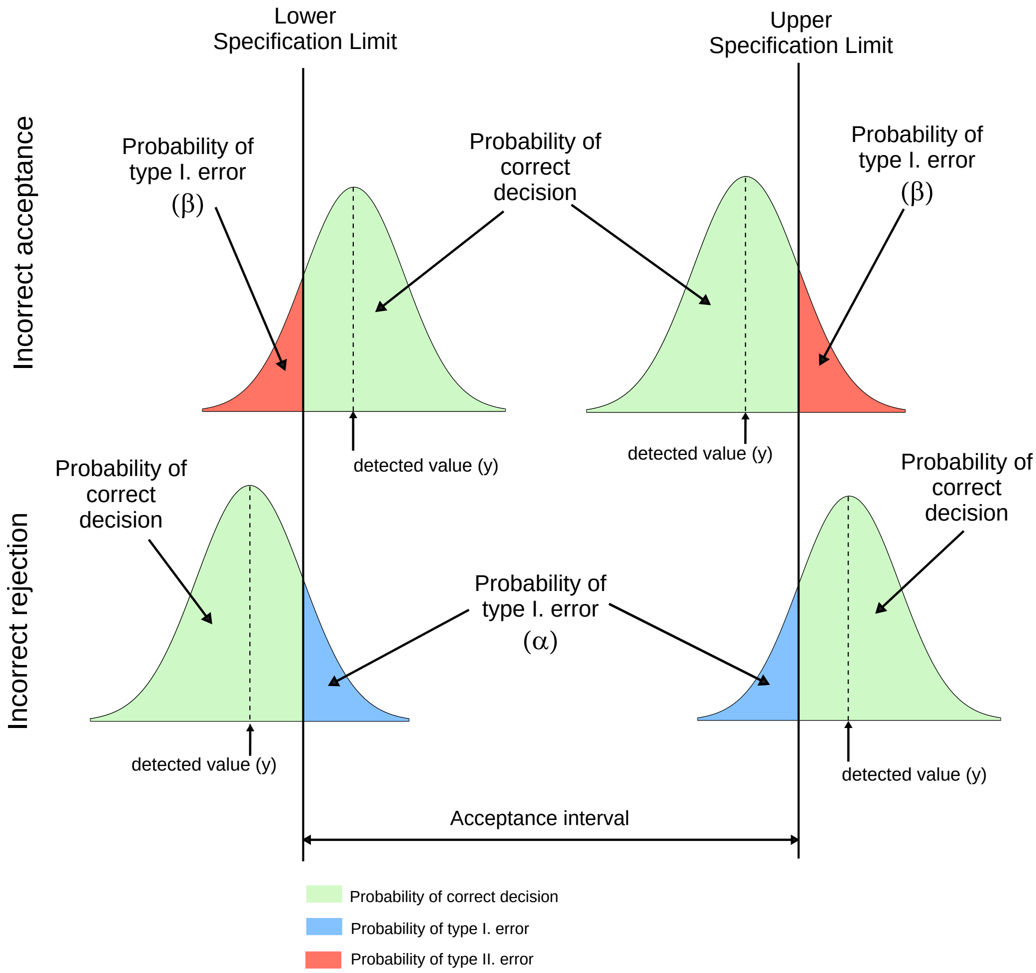

FIGURE 1.1: Illustration of measurement errors on conformity assessment (source: own edition based on AIAG, 2010)

On Figure 1.1, y denotes the observed value of the monitored product characteristic. Due to the existence of measurement error, the real value can be considered as a probabilistic variable which is assumed to follow normal distribution in this example. If the observed value is close to the specification limit, the probability of incorrect decision increases. Incorrect acceptance (type II. error denoted by β) is committed if the product is conforming based on the observed value however, the real value falls outside the acceptance interval. In the opposite case, incorrect rejection (type I. error denoted by α) occurs, that is to say the observed value falls outside the acceptance interval but the product is conforming based on the real product characteristic.

As it was mentioned before, measurement errors not only affect the outcome of conformity testing but also can have significant impact on statistical process control. The effect represented by Figure 1.1 can be applied to statistical control charts as well (Figure 1.2):

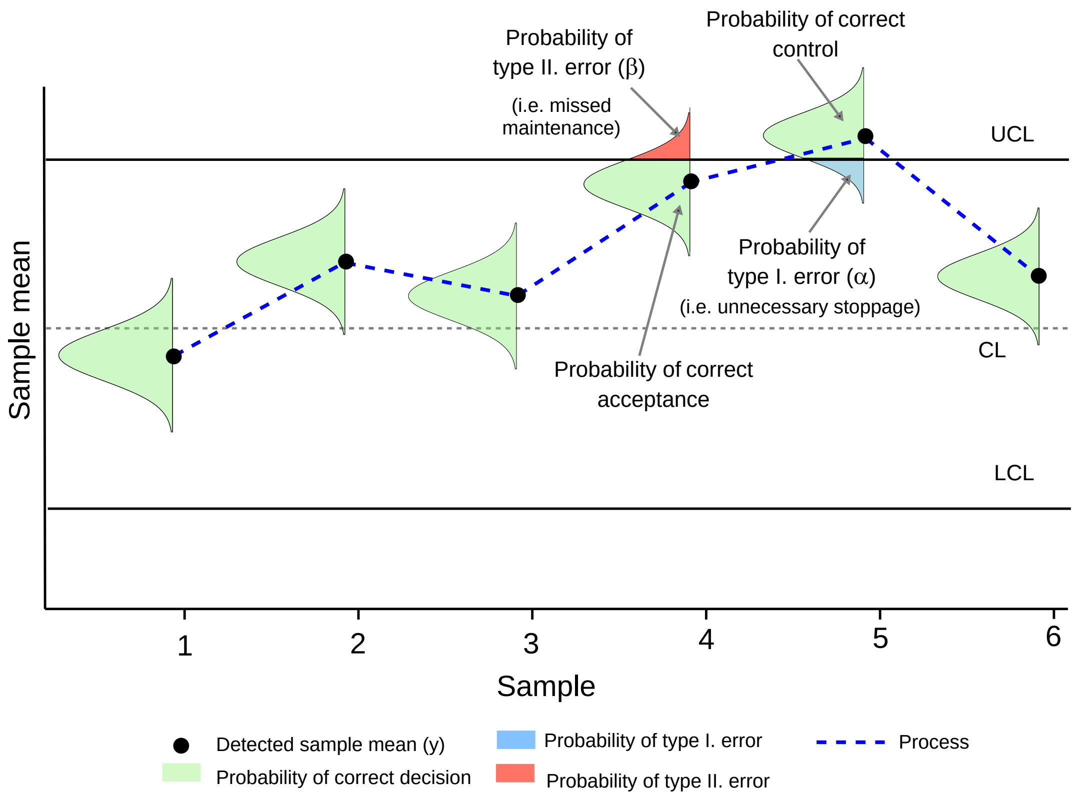

FIGURE 1.2: Illustration of measurement errors on control charts

Figure 1.2 shows an $\overline{X}$ chart where the observed sample mean is denoted by the black dot and real sample mean is represented as probabilistic variable (It is assumed that measurement errors follow normal distribution.). The fourth sampling event highlights the probability of incorrect acceptance of the process. If type II. error is committed, a necessary maintenance is skipped which can lead to delayed detection of process shift or even to serious machine failure. Fifth sampling event shows the probability of type I. error leading to unnecessary stoppage which can be extremely cost-intensive too.

In order to reduce the decision risks, traditional control charts needs to be improved and risk-based aspect (RB) needs to be considered, where control limits are optimized in order to minimize the risks of the decisions. Although there are recommendations by several measurement manuals (BIPM et al., 1995, Eurachem, 2007b, they cannot handle the measurement uncertainty comprehensively, because these recommendations assume the normality of the measurement error distribution. The literature of statistical process control includes a wide scale of control charts operating on reliability base, but a gap can be observed in the literature according to the field of the control charts based on risk-based philosophy. The aim of the thesis is to develop a family of risk-based control chart which is able to reduce the decision risks arising from the measurement uncertainty. In my thesis I determine the following research questions and research proposals:

Research Questions

Q1: Which moments of the measurement error distribution function (expected value, standard deviation, skewness, kurtosis) can describe the measurement uncertainty?

Q2: What is the cost reduction rate that can be achieved while using a risk-based multivariate control chart instead of a traditional multivariate chart?

Q3: What is the cost reduction rate that can be achieved while using a risk-based control chart with variable sample size and sampling interval compared to the traditional VSSI X-bar chart?

Research Proposals

P1: All four moments of the measurement error distribution function need to be considered by the characterization of the measurement uncertainty.

P2: 3-5% total decision cost reduction can be achieved with risk-based multivariate control chart compared to the "traditional" multivariate control chart.

P3: 3-5% total decision cost reduction can be achieved with risk-based adaptive control chart compared to the "traditional" adaptive control chart.

Rest of the dissertation is organized as follows:
In Chapter 2, I introduce the methodology and results of the systematic literature review, Chapter 3 presents the proposed methods. Simulation results are provided in Chapter 4, sensitivity analyses are conducted in Chapter 5, applicability is demonstrated through real practical examples by Chapter 6. Finally, I summarize my research results and implications in Chapter 7.
The next chapter reviews the scientific literature related to the research topic.

Chapter 2

Related Studies

2.1 Methodology of the literature research procedure

In order to explore the related studies and review the most relevant researches according to the field of statistical control charts/measurement uncertainty, I conducted a systematic review including classification of the related articles as well. A survey-based content analysis was applied to classify the related researches (see Kolbe and Burnett (1991) for more detailed information about content analysis). Following the structure used by Maleki et al. (2016) and Hachicha and Ghorbel (2012) - who conducted content analysis in the field of statistical process control - the literature research included two steps: First, the set of appropriate scientific studies needs to be determined. Secondly, the identified set of papers needs to be classified using predefined categories.

In order to ensure that relevant studies were not missed, I also extended the aforementioned two steps with an additional one: Refinement. Within this step, citation data of the collected papers were also analyzed. With the help of this, I was able to find those papers that were not included by the current platform I used for the search. Based on that, the structure of systematic literature search can be described as follows:

1. Collection
2. Classification
3. Refinement

Since my research questions cover two main research fields (research question Q1 refers to measurement uncertainty researches and research questions Q2-Q3 apply to the research field of statistical control charts), the aforementioned literature search was provided twice: on one hand, I considered the set of researches dealing with measurement uncertainty, on the other hand the relevant literature of control charts was analyzed. In Subsection 2.1.1 and 2.1.2, the two major steps of the literature search are discussed.

2.1.1 Collection

In order to determine the appropriate set of literature, scientific journal articles, industrial standards and conference papers were considered using computerized search with specific keywords like: "*measurement uncertainty*","*measurement error*", "*gauge error*", "*skewed distribution*" to find the researches related to measurement uncertainty topic. Furthermore, "*control chart*", "*statistical process control*", "*variable monitoring*" terms were used to find the researches associated with control chart topic.

The main platforms and publishers I used for the literature search were Google Scholar, ScienceDirect, Web of Science, Taylor & Francis, Springer Link, Emerald Insight, IEEE Xplore, Scopus and IOPScience.

It is also important to define which papers are relevant according to the topic of my thesis. Since the research questions/proposals cover two main research fields, relevance of the papers needs to be defined from the perspective of the two research fields:

Control charts The paper is relevant and can be added to the collection if it develops a new methodology related to control charts. That is to say, studies that are focusing on existing control chart approach on a new field of application are not relevant from the point of view of the thesis.

Measurement uncertainty and conformity control The paper (or standard) is relevant if it either deals with the expression/interpretation of the measurement uncertainty under symmetric/asymmetric measurement error distribution or focuses on the effects/treatment of measurement uncertainty in conformity control.

2.1.2 Classification

As the next step, the collected papers were classified using a conceptual classification scheme. Similarly to Maleki et al. (2016) and Hachicha and Ghorbel (2012) I used predefined questions and possible answers to classify the selected papers. Since the research questions cover two main research fields (measurement uncertainty and statistical process control charts), two versions of classification surveys were constructed. First, I introduce the survey used for the classification of literatures regarding measurement uncertainty.

Literature of measurement uncertainty

Table 2.1 shows the question-response set regarding the papers dealing with measurement uncertainty.

TABLE 2.1: List of questions and possible responses according to the papers with measurement uncertainty topic

Nr.	**Questions/Responses**
1	What is the main focus of the paper?
1.1	*It deals with the expression of measurement uncertainty.*
1.2	*It deals with the consequences of measurement uncertainty in product conformity.*
2	What kind of distribution the paper assumes for the measurement errors?
2.1	*It assumes symmetric error distribution.*
2.2	*It assumes asymmetric error distribution.*

My thesis focuses on statistical process control and conformity control, therefore it is important to identify which studies develop/discuss different approaches for the expression of the measurement uncertainty under different conditions, and which studies aim to propose methods for handling of measurement uncertainty in conformity control. While the first set of studies provides different approaches to determine or describe measurement uncertainty (not just in conformity control), the second set of researches focuses more on the consequences of measurement uncertainty.

Since Research Question Q1 applies to the influence of the moments of the measurement error distribution, it is also important to identify the related studies that considered asymmetric distribution types. In this thesis, the most relevant studies are the ones that:

- deal with the consequences of the measurement uncertainty in conformity control and/or
- assume asymmetric distribution type(s) for the measurement error.

In the next part, I introduce the questions/responses related to the literature of control charts.

Literature of control charts

Table 2.2 shows the question-response set regarding the papers proposing control charts.

TABLE 2.2: List of questions and possible responses according to the papers developing control charts

Nr.	Questions/Responses
1	What is the dimension of the monitored quality characteristic(s)?
1.1	*Univariate*
1.2	*Multivariate*
2	Does the proposed method consider measurement errors?
2.1	*No (Traditional control chart)*
2.2	*Yes (Risk-based control chart)*
3	Is the proposed control chart applicable under non-normality?
3.1	*No (Parametric control chart)*
3.2	*Yes (Non-Parametric control chart)*
4	Does the proposed control chart apply variable chart parameters?
4.1	*No, it is a control chart with fixed parameters (FP chart)*
4.2	*Yes, it is an adaptive control chart*
5	If the study deals with adaptive chart parameters, which parameters are variable?
5.1	*Sample Size (VSS control chart)*
5.2	*Sampling Interval (VSI control chart)*
5.3	*Control Limits (VSL control chart)*
5.4	*All the three chart parameters (VP control chart)*

In this literature research, control charts for attributes and economic design-related papers were not considered. The first question identifies whether the given paper deals with univariate (e.g., Shewhart type, EWMA, MA, CUSUM) or multivariate quality characteristics (e.g., T^2, MCUSUM, MEWMA). Question 2 is intended to reveal the nature of the proposed approach from risk's point of view. To be more specific, studies that consider measurement uncertainty as a risk factor during the control procedure were labeled as "Risk-based" approaches. Similarly, if the given paper proposes a new type of control chart however, it does not take the measurement uncertainty into account, it was labeled as "Traditional" approach. Question 3 makes difference between studies dealing with parametric and non-parametric control charts, and question 4 distinguishes between adaptive control charts and control charts with fixed parameters. Question 5 only makes sense if response 4.2 is true. It is necessary to note, that responses 5.1, 5.2 and 5.3 are allowed to be true in the same time by one study, meaning that papers can be assigned to VSS, VSI and VSL categories simultaneously. However, since VP charts are getting increased attendance, this set of researches/papers was considered as a separated group (Nenes, 2011).

2.1.3 Refinement

There are situations when the currently used platform does not include a paper that would be relevant according to the selected topic. So that a relevant paper may not be missed, I analyzed the list of citing references for all the papers found by step 1 (collection). If further papers were found (through the citation list), they were added to the already collected researches.

Not only the selected articles but the information about citation was also gathered. The information were stored in a database with two tables where the first one contains the information about the scientific paper (authors, title, journal, keyword, topic) and the second one represents the citing relations between the collected articles. This structure allows to build network-type visualizations in order to illustrate the structure of the studied research areas (and their sub-areas as well.).

It is important to note, that PRISMA Statement (Preferred Reporting Items for Systematic reviews and Meta-Analyses) was developed in order to support the procedure of systematic literature review. PRISMA provides a checklist with 27 elements and flow-diagram to help the researchers to improve the quality of literature review (Moher et al., 2009). The referred flow-diagram includes the following main steps:

1. Identification: Collection from databases and other external sources
2. Screening: Removal of duplicates
3. Eligibility checking
4. Removal of not relevant studies

The main difference between the applied literature research methodology and PRISMA approach is the following: While PRISMA provides a simple process flow, the proposed literature research method includes iterative elements. Even the classification rules can be revised and refined if new research field can be identified through the exploration of the citation data.

In the following section I introduce the result of the literature research.

2.2 Result of the literature research

Networks were used to visualize the quantity of researches and citing relationship in the analyzed fields. In this section I introduce two main networks, the first represents the result of the literature research in measurement uncertainty and conformity control area, while the second visualizes the structure of papers regarding control charts.

2.2.1 Measurement Error and Conformity Control

Figure 2.1 shows the result of the systematic literature search I conducted related to measurement uncertainty and conformity control.

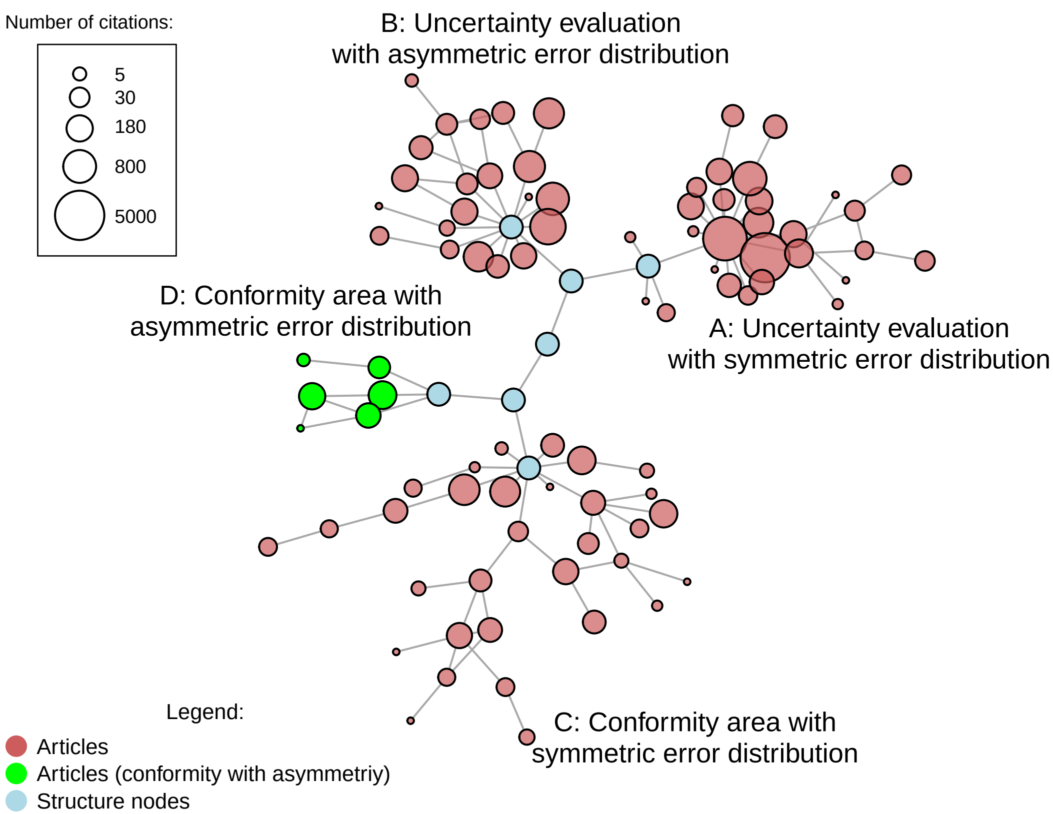

FIGURE 2.1: Result of the literature research (measurement uncertainty area)

Since my first research question (Q1) is related to the distribution properties of measurement error, the main goal of this review was to identify the most relevant studies that deal with asymmetric measurement error distributions in the field of conformity control. After the search I classified the papers using the survey described by Table 2.1.

Nodes represent the reviewed papers and edges represent the citing relationship between them. Blue nodes illustrates the responses related to each question from Table 2.1 so they illustrate the result of the classification (in other words, the blue nodes represent the structure of the aforementioned survey). The reviewed and classified papers were colored with red, however there is a group of papers highlighted with green. I highlighted those nodes, because that group includes papers considering asymmetric measurement error with the aspect of conformity control (group D). In addition the size of the nodes represents the citation numbers (How many times they were cited by others.) in logarithmic scale.

86 studies were selected and categorized and 6 papers out of the 86 were classified into group "D" (colored with green). Although many researches focused on the evaluation of measurement uncertainty even assuming asymmetric measurement error distributions, only a few considered the effect of asymmetric measurement uncertainty and its consequences in conformity control. I summarize the most relevant contributions in two steps, starting with groups "A", "B" and "C".

Groups "A", "B" and "C":

As part of the Six Sigma approach, measurement system analysis (MSA) and

R&R tests (repeatability and reproducibility) are often used to evaluate measurement uncertainty of a measuring device. These methods are useful to get knowledge about the performance of measurement device or system, however their purpose is to support the decision making about the validation of the device/system and do not consider the further consequences of the measurement uncertainty (AIAG, 2010).

In 1995, the attitudes were changed related to the measurement uncertainty with the construction of the Guide to the Expression of Uncertainty in Measurement (GUM)(BIPM et al., 1995). GUM proposes the expression of the measurement uncertainty in two ways. On one hand, the measurement uncertainty is expressed as a probability distribution derived from the measurement. On the other hand, this uncertainty can be described as an interval. In the first case, the standard deviation is used for the characterization of the distribution (standard uncertainty). If the result of the measurement is obtained by combining the standard deviation of several input estimates, the standard deviation is called combined standard uncertainty. In the second case, the length of the interval can be determined by the multiplication of the combined standard uncertainty and a coverage factor k and called as expanded uncertainty. There are several guidelines that proposes 2 as a value of the coverage factor k (BIPM et al., 1995, Eurachem, 2007a, Heping and Xiangqian, 2009, Rabinovich, 2006, Jones and Schoonover, 2002), producing 95.45 % confidence level, however this statement is only true if the combined uncertainty follows normal distribution, otherwise the estimation of the confidence level is not correct (Vilbaste et al., 2010, Synek, 2006).

Asymmetry of the distribution can also lead to incorrect estimation and incorrect decisions as well. The JCGM Guide 101 (BIPM et al., 1995) introduces that exponential and gamma distributions are observable as asymmetric examples, furthermore, researches have shown that asymmetry can appear in combined standard uncertainty as well (Herrador and Gonzalez, 2004, D'Agostini, 2004, Pendrill, 2014). Not only skewness can be the root cause of the over- or underestimation of confidence level. Kurtosis can also vary by different measurement devices or systems. Leptocurtic and platykurtic distributions are also observable by several measurement systems (Martens, 2002, Pavlovcic et al., 2009).

If the measurement error distribution follows non-normal distribution, the confidence level will be estimated incorrectly and using the k=2 proposal, decision errors can be made, since the principal assumption of the proposal is not valid. Furthermore, the rules based on the assumption of normal distribution do not consider the consequences of the decision errors, however they can lead to considerable problems.

Group "D":

Figure 2.2 shows the structure of the papers classified in group "D" in details.

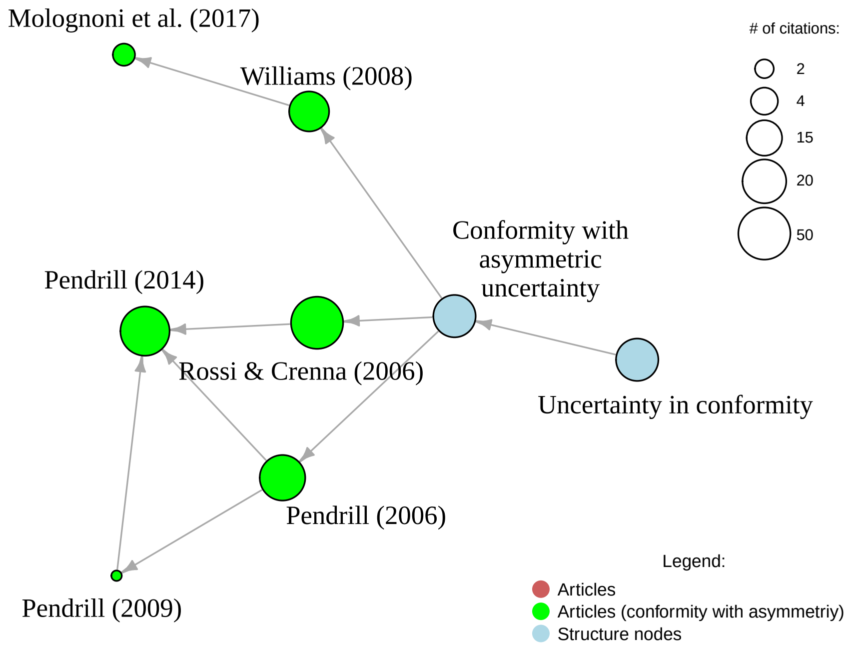

FIGURE 2.2: Result of the literature research (measurement uncertainty area-Sub-graph)

There were also researches taking measurement uncertainty and the decision consequences into account. Rossi and Crenna showed that measurement uncertainty should not be treated as an interval or a simple standard deviation, but needs to be considered as a probability distribution in order to avoid incorrect decisions (Rossi and Crenna, 2006). Williams (2008) pointed out that decision rules must be carefully defined when skewed measurement error distribution is assumed. Although, Forbes has proposed a method treating the conformance assessment as a Bayesian decision, he only considered the cost and revenue of the incorrect decisions (Forbes, 2006). Later, Pendrill has developed a more comprehensive model considering measurement uncertainty in conformity sampling. The model included all the four decision outcomes (correct acceptance, false rejection, false acceptance and correct rejection) however, only correct decision-, and testing costs were considered during the calculations (Pendrill, 2008, Pendrill, 2014).

The referenced papers made steps towards the risk-based aspect of the conformity control, they did not consider all the four decision outcomes in the calculations and however, they also considered even asymmetric measurement error distributions, the strength of the characteristics of the measurement error distribution (skewness, kurtosis) were not analyzed. Research Question Q1 is still valid after the literature review, since I did not find any paper that answered the question and investigated how 3^{rd} and 4^{th} moments of measurement error distribution affects the decision outcomes during conformity control. In my thesis, I develop a risk-based model in the statistical process control including all the four decision outcomes and examine the impact of 3^{rd} and 4^{th} moments of the measurement error distribution.

2.2.2 Measurement Error and Control Charts

The same literature search approach was conducted in order to explore the most relevant studies of control charts. The result of the systematic review is introduced by Figure 2.3.

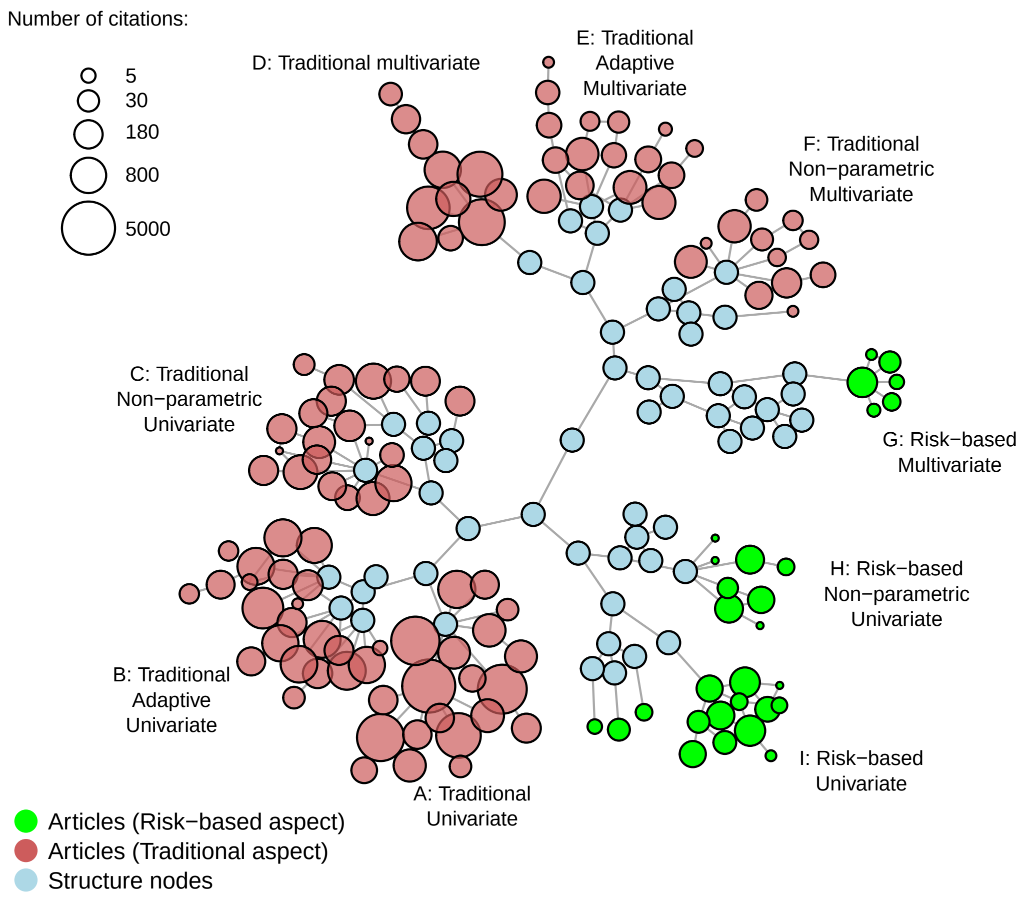

FIGURE 2.3: Result of the literature research (Control chart design research area)

On Figure 2.3, the logic of coloring remained the same: blue nodes represent the structure of the control charts defined by the predefined survey, red nodes denote the reviewed articles regarding traditional control charts (without considering measurement errors). Papers that developed control charts considering the effect of measurement error were highlighted with green (risk-based aspect), and node size represents citation numbers.

First, I summarize the most relevant papers in the field of traditional control charts.

Traditional control charts (Group "A-F"):

The first statistical control chart was developed by W.A. Shewhart in 1924 (Shewhart, 1924) to monitor the process expected value. The process is labeled to "in-control" if the sample mean falls within the Lower and Upper Control Limits (LCL and UCL) (Shewhart, 1931). Although the X-bar chart was able to detect when the expected value of the process changes significantly, its main deficiency is the inability to detect small shifts. In order to rectify that, CUSUM (Cumulative Sum) and EWMA (Exponential Weighted Moving Average) control charts were proposed (Page, 1954, Roberts, 1959). However univariate control charts were powerful tools to detect process shift, they were not able to monitor more than one product characteristics simultaneously. Though Shewhart dealt with monitoring of more correlated characteristics, the multivariate control chart has its origins in the research of H.

Hotelling, who developed the T^2 chart based on Student's t-distribution (Hotelling, 1947). Subsequently, other multivariate control charts were developed like multivariate sum (MCUSUM) control chart (Crosier, 1988, Pignatiello and Runger, 1990), and the exponentially weighted moving average chart (MEWMA), developed by Lowry and Woodall (Lowry et al., 1992). Several references give more detailed discussion about the multivariate quality control reviewed by Jackson (1985).

Multivariate and univariate control charts were commonly used for process control, however, their application condition is the preliminary knowledge of the distribution of the controlled product characteristic(s). Most control charts assume normal distribution or a known form of a particular distribution for the monitored product characteristic (Yang et al., 2011). For the elimination of the problem, several researches developed nonparametric control chart approaches (see: Bakir and Reynolds, 1979; Amin et al., 1995; Bakir, 2004; Bakir, 2006; Chakraborti and Graham, 2008 for univariate charts and Chakraborti et al., 2001; Bakir, 2006, Tuerhong et al., 2014; Chakraborti et al., 2004 for multivariate control charts).

The evolution and complexity of production processes resulted in the development of more flexible control charts with adaptive control chart parameters (n, h, k). If the monitored process is "in-control" state, smaller sample size, longer sampling interval and wider accepting interval are used. However, in "out-of-control" the adaptive charts apply stricter control policy (larger sample size, shorter sampling interval, and narrower accepting interval) (Lim et al., 2015).

Reynolds, Amin, Arnold and Nachlas were the first who developed an X-bar chart with variable sampling interval (VSI) (Reynolds et al., 1988), and their research inspired a number of researchers opening the research field of adaptive control charts. (Runger and Pignatiello, 1991, Chew et al., 2015, Naderkhani and Makis, 2016, Bai and Lee, 1998, Chen, 2004). Subsequently Prabhu, Runger and Keats developed an X-bar chart with variable sample size (VSS) (Prabhu et al., 1993) followed by several improvements (Costa, 1994, Tagaras, 1998, Chen, 2004). As a further contribution to the field, VSSI control charts were developed (variable sample size and sampling interval) where sample size and sampling interval are modified simultaneously (Costa, 1997, Costa, 1998, Costa, 1999, Chen et al., 2007, De Magalhães et al., 2009).

In order to determine the optimal parameter levels for the adaptive control charts, numerous studies aimed to apply economic design methodology minimizing the average hourly cost during the process control (Lee et al., 2012, Lin et al., 2009, Chen et al., 2007, Chen, 2004).

During the literature research, I also reviewed the domestic literature and it is observable that Hungarian control chart articles and studies are rather descriptive and just a few research focused on development.

Risk-based control charts (Group "G-I"):

Producers' and suppliers' risks are frequently discussed topics in the field of conformity or process control (see e.g.: Lira, 1999). Risks can arise from different sources, such as uncertainty in the real process parameters or imprecision of the measuring device. Lack of knowledge regarding the real value of the process parameters or imprecision of the measuring device can be considered as uncertainty during the application of control charts. Several studies showed that parameter estimation has a significant impact on the performance of control charts (Jensen et al., 2006, Zhou, 2017). On the other hand, measurement errors can lead to incorrect decisions and increases the number of type I. and type II. errors (Pendrill, 2008).

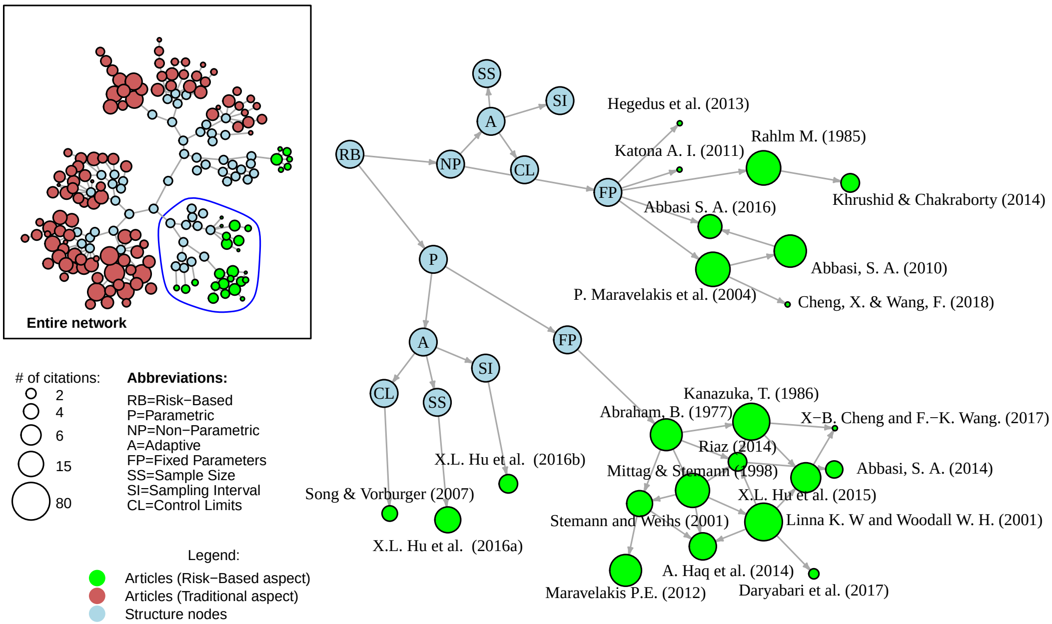

FIGURE 2.4: Result of the literature research (Control chart design research area - Univariate subgraph)

In 1977, Abraham studied the performance of $\overline{X}$ chart by adding measurement error to the original process. His research inspired several scholars and opened the way for studies with the aim to analyze the effect of measurement error on control charts (Abraham, 1977).

As a contribution to the topic, Kanazuka showed that relatively large measurement error reduces the power of $\overline{X}$-R charts and proposed increased sample size to improve the performance (Kanazuka, 1986). Later Mittag and Stemann examined the impact of gauge imprecision on the performance of $\overline{X}$-S control charts (Mittag and Stemann, 1998). Based on this study, Linna and Woodall further developed a measurement error model with covariates and investigated how measurement error (based on the referred model) influences the performance of X and S^2 charts. Several studies adopted this model and investigated the performance of different types of control charts under the presence of measurement error while assuming linearly increasing variance (Haq et al., 2015, Hu et al., 2015, Hu et al., 2016a, Maleki et al., 2016, Maravelakis et al., 2004, Maravelakis, 2012).

There were economic design researches considering the effect of measurement errors as well, however they mainly focused on control charts with fixed parameters. Rahlm investigated how non-normality and measurement error influences the economic design regarding X-bar control chart (Rahlm, 1985). This research was extended to asymmetric X and S charts by Yang, 2002. Additional studies proposed economical design method for memory-based control charts as well, such as exponentially weighted moving average (EWMA) chart based on measurement error (Saghaei et al., 2014, Abbasi, 2016).

Although, several studies investigated the performance of Shewhart control charts under the presence of measurement error, only a few have dealt with the measurement uncertainty related to the adaptive control charts. Hu et al. (2016b) developed VSS X-bar chart considering the effect of measurement error using linear covariate model. The same scholars later extended their research with the design of VSI X-bar control chart (Hu et al., 2016a).

The aforementioned studies have considered the measurement error by the application of the control chart and analyzed the effect of measurement error on process control effectiveness but they did not take the decision outcomes (consequences) into account. Although, I have dealt with risk-based control chart development minimizing the overall cost of decision outcomes in my former researches and contributions (Hegedűs et al., 2013b, Katona et al., 2014), these articles focused on $\overline{X}$, MA and EWMA charts. In my thesis, I develop multivariate and adaptive control charts with the consideration of measurement error and the consequences of decisions in order to further extend the family of the risk-based control charts.

Not only univariate but the field of multivariate control charts was also reviewed during the literature search. Figure 2.5 shows the sub-graph of the risk-based multivariate control charts.

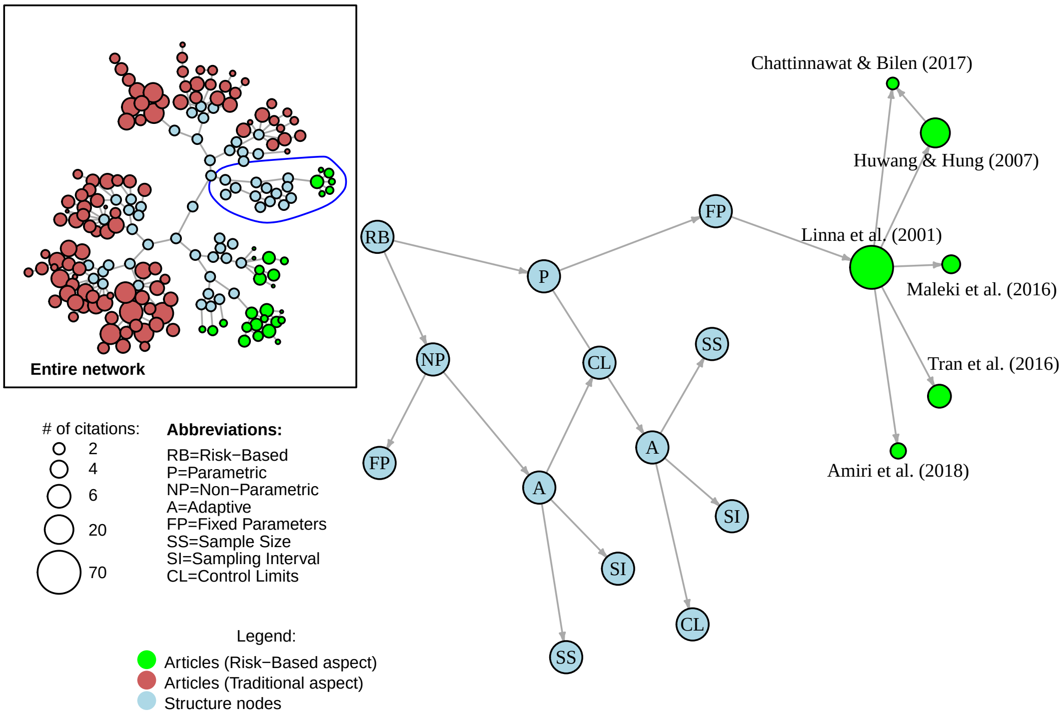

FIGURE 2.5: Result of the literature research (Control chart design research area - Multivariate subgraph)

Measurement error can reduce the power of control charts even in multivariate process control. Linna et al. (2001) investigated the performance of χ^2 chart under the presence of measurement error and their study has inspired several researchers. Huwang and Hung (2007) and Amiri et al. (2018) investigated the effect of measurement errors on the monitoring of multivariate measurement variability. In 2016, Maleki et al. (2016) used extended multiple measurement approach in order to reduce the effect of measurement error on ELR control chart while monitoring process mean vector and covariance matrix simultaneously. Performance of the Shewhart-RZ chart was examined under the presence of measurement error by Tran et al. (2016). Furthermore, Chattinnawat and Bilen concluded that measurement error leads to inferior performance of the Hotelling's T^2 chart. Their study helps the practitioners to predict how T^2 will behave with respect to the precision of the gauge (i.e. %GRR).

Similarly to the univariate area, the aforementioned studies focused on the effect of measurement errors on control chart performance, however they did not consider the decision outcomes or the risk of incorrect decisions due to the measurement error. After the literature review, Research Questions Q2 and Q3 are still valid because no study was found that develops multivariate or adaptive control chart with the consideration of measurement uncertainty and consequences of the correct/incorrect decisions as well.

It is necessary to note that uncertainties can relate to the system parameters (parametric uncertainty) and they can arise due to the modeling of complex systems (nonparametric uncertainties) (Adhikari, 2007, Pokorádi, 2008, Pokorádi, 2009). Although there are solutions for the modeling of nonparametric uncertainties in engineering science (Oberkampf et al., 2002, Adhikari et al., 2007, Helton et al., 2007), the research area of control charts considers the effect of measurement error as parametric uncertainty.

The results of the systematic literature review were analyzed for both research areas separately, however it is also valuable to determine how strong is the "linkage" between the research areas of control charts and measurement uncertainty. Subsection 2.2.3 introduces the citation relationships between the two networks.

2.2.3 Citations between the two networks

In order to analyze the "linkage" between the two networks, I also examined all the citation data to find those papers that were cited by studies from the other research field (network). Assume that paper "A" as part of the measurement uncertainty area is cited by paper "B" that develops a new control chart. In this case, their relationship is highlighted by an additional edge between them, indicating that they establish connection between "control chart" and "measurement uncertainty" research areas. Figure 2.6 illustrates the citations between the two networks.

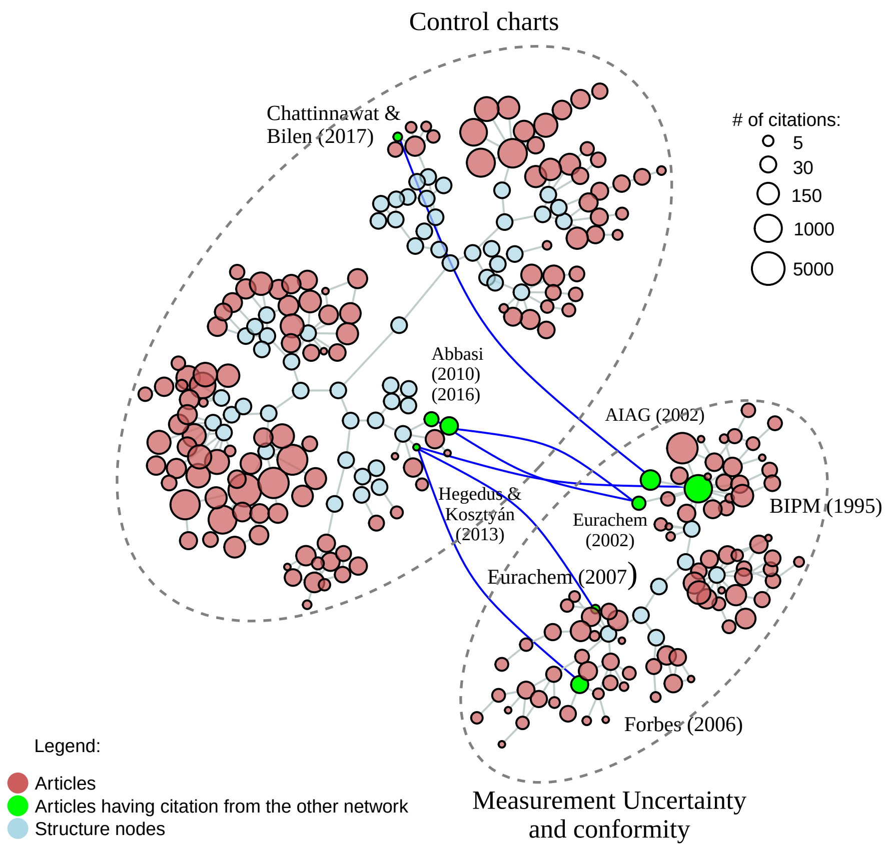

FIGURE 2.6: Citations between the two networks

The citations between the two research areas are denoted by blue edges, and newly connected nodes are highlighted with green. As Figure 2.6 shows, only 9 papers and 6 citations could be found between the networks based on the conditions described above. Although there are studies developing control chart under the presence of measurement error, only few of them utilize the results given by researches related to measurement uncertainty and conformity control area.

The findings also confirm the importance of my research from connection's point of view. As an additional contribution, my work also aims to strengthen "linkage" between the two research fields by developing the aforementioned risk-based methods that can be used in control chart design and conformity control.

In the previous subsections, I showed the structure of control chart and measurement uncertainty research areas. Networks can illustrate the gaps and commonly studied sub-areas very well however, as a weak point, they cannot show the research trends according to time.

To overcome this issue, in Subsection 2.2.4, I introduce the most important "milestones" or research results based on the year of their publication.

2.2.4 Analysis of research trends

On Figure 2.7, most important "milestones" or research results were placed onto a time line. Parallel research sub-areas are presented by multiple horizontal lines. If a research or study develops a new concept, new sub-area is also created and it is represented by an additional path on Figure 2.7. For example, in 1947, Hotelling developed multivariate control chart and opened the way of multivariate quality control which is denoted by an additional horizontal line.

The circles are representing publications moreover, specific papers dealing with control chart performance under the presence of measurement error are highlighted with green. It is necessary to note that of course, more additional research directions could be identified based on different aspects (or categorization rules) (e.g., economic design researches, parameter estimation, etc...). In the interest of transparency, in this analysis, I use the same logic for the categorization of papers as it was introduced by Subsection 2.1.2. In other words, parallel horizontal lines represent the evolution of multivariate and adaptive control charts or measurement uncertainty evaluation / conformity control researches, while other important areas (like nonparametric chart design or measurement uncertainty evaluation based on moments) are mentioned in the discussion below.

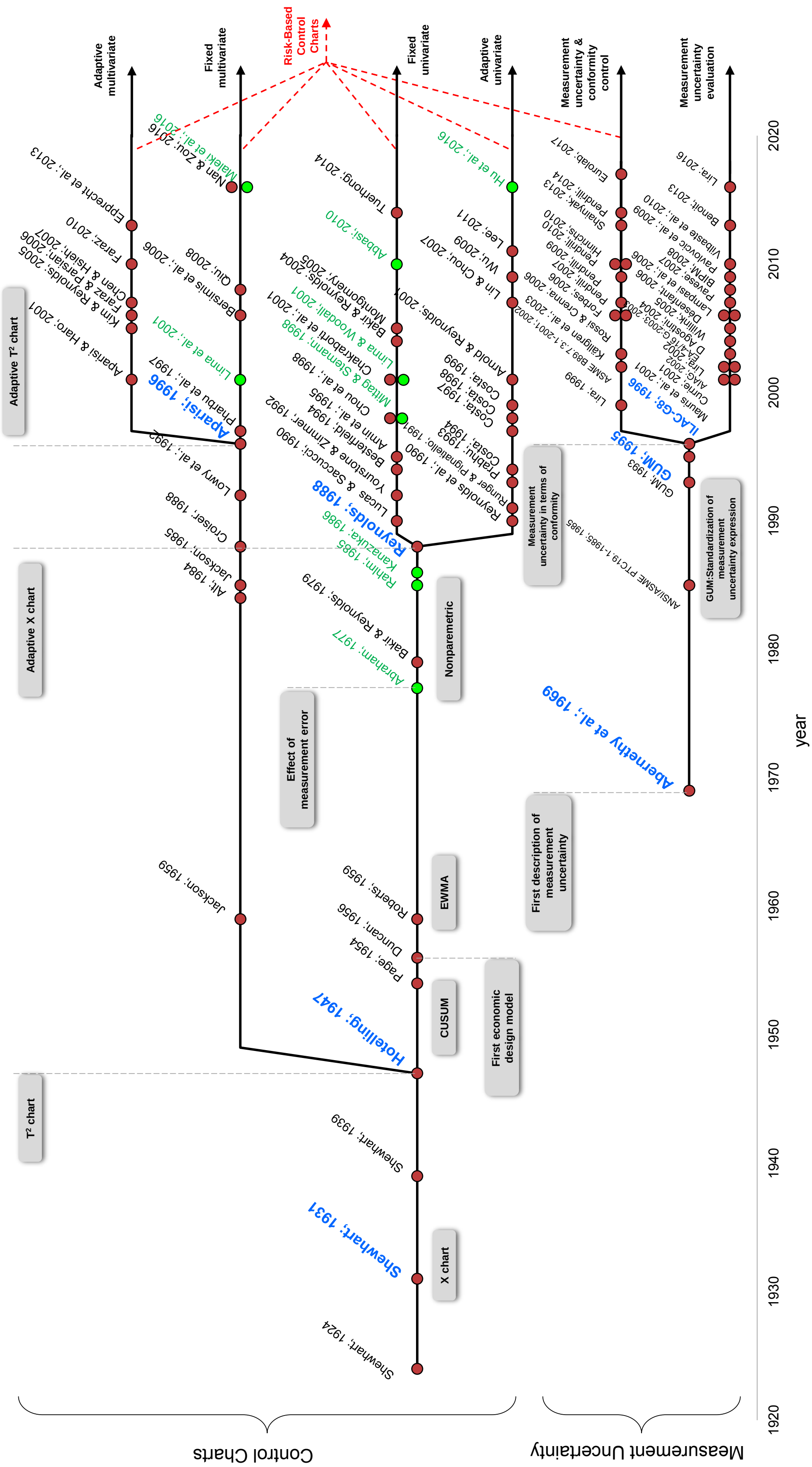

FIGURE 2.7: Most important "milestones" in control chart and measurement uncertainty research

Control Charts The research of statistical control charts has its origins in 1924 (Shewhart, 1924) and the first univariate control chart was developed by Shewhart, 1931. Monitoring ability of control charts was extended by Hotelling (1947) who introduced multivariate control procedure. Control of multiple product characteristics was brand new idea however, it did not get outstanding attendance at this time.

As improvement of X-bar chart, CUSUM and EWMA schemes were designed to improve the detection power of small shifts. The first economic design model was described in 1956 (Duncan, 1956) inspiring numerous scholars to extend this methodology to the different type of control charts as well. Economic design became substantial research direction in both, univariate and multivariate fields.

Growth and diversity of production environment required the ability of adaptation to any conditions of the manufacturing process which led to the design nonparametric control charts (Bakir and Reynolds, 1979). After that point, many researches aimed to extend this methodology to different type of control charts.

The next decisive result was the first adaptive univariate control chart with variable sampling interval developed by Reynolds et al. (1988). In parallel, multivariate control charts were getting increased attendance especially after the development of the first multivariate adaptive control chart (Aparisi, 1996). The next important contribution was the development of control charts with variable sample size of sampling interval.

Nowadays we have large scale of univariate and multivariate control charts with adaptive or fixed parameters. Outstanding research topics are: robust design of nonparametric control charts, economic design and pattern recognition. In order to confirm my findings regarding the control chart design research time line, I conducted text mining, based on Google Scholar database. The analysis includes the following steps:

1. Scientific paper titles (and additional data) containing "control chart" term were collected from Google Scholar search in 5 year-long time intervals (starting with 1990)

2. Collected Google Scholar data were preprocessed using R's "*tm*" package (stopwords removal, transform to lowercase, etc...).

3. Term frequencies were calculated for each time interval (disregarding "*control chart*" terms within titles).

4. Wordclouds were provided regarding each time interval.

The wordclouds show the most frequently used terms in paper titles related to control chart research area from 1990 to 2018, illustrating how "hot topics" changed over time. In the wordclouds, red color denotes the terms that strengthened and blue color highlights those ones that weakened compared to previous period (based on the changes in frequency values) (Figure 2.8).

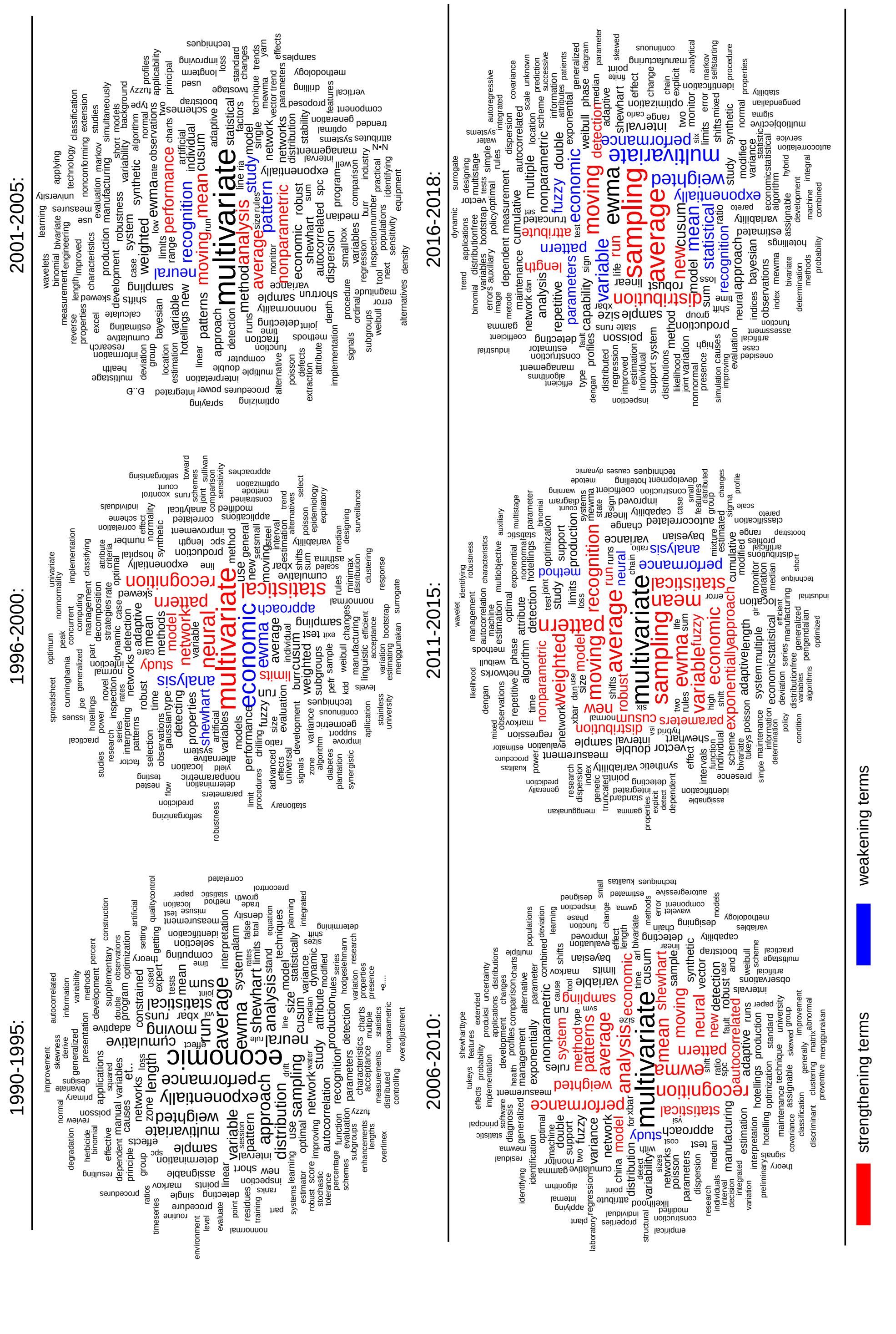

FIGURE 2.8: Most frequent terms in control chart paper titles (from 1990 to 2018)

Based on Figure 2.8, it is clearly visible that economic design was the dominant aspect in control chart design, however, its emphasis decreased after 1995. In parallel, multivariate control chart design became the most important research area. The next rising topics were pattern recognition, nonparametric approaches and robust design of control charts but multivariate aspect kept its leading role. (2001-2015).

After 2015, control charts with variable parameters (sample size, sampling interval) and sampling strategies became the most significant topics taking the place of multivariate process control.

This additional text mining-based analysis (considering approximately 4000 papers) also confirms the conclusions of Figure 2.7.

Measurement uncertainty and conformity control Most of the aforementioned control charts do not take the measurement uncertainty into account however, its effect and importance on measurement results were showed by several scholars. The first measurement uncertainty model was described by Abernethy et al. (1969). Later, a comprehensive international standard was provided by ISO organization: Guide to Expression of Uncertainty in Measurement (GUM) (BIPM et al., 1993).

In 1996, International Laboratory Accreditation Cooperation was provided guidelines on assessing conformity in terms of measurement uncertainty (ILAC, 1996), which inspired several researchers to investigate the effect of measurement uncertainty on conformity control. The main stream was divided into two areas: treatment of measurement uncertainty in conformity control and measurement uncertainty evaluation.

Consumer's and producer's risk became outstanding in conformity control, moreover, Pendrill's researches were pioneer because they provided improved conformity control approaches under the presence of measurement uncertainty (Pendrill, 2006, Pendrill, 2007, Pendrill, 2008, Pendrill, 2009, Pendrill, 2010).

In the other stream, several scholars showed that measurement uncertainty should be treated as probability distribution and not just as an interval. They introduced new methodologies to express measurement uncertainty under asymmetric measurement error distributions (Herrador and Gonzalez, 2004, Synek, 2007, Pavlovcic et al., 2009, D'Agostini, 2004). Although that was significant contribution to measurement uncertainty area, only a few researchers applied the concept of asymmetric measurement uncertainty in conformity control studies (as it was shown by Figure 2.2).

Common points of the two areas The appearance of measurement uncertainty studies have been inspired researchers to investigate the effect of measurement error on control charts. The articles considering the effect of measurement errors are denoted by green circles on Figure 2.7.

The first study regarding measurement error and control charts was conducted in 1977 (Abraham, 1977) however, the number of these papers is way below the quantity of publications from other control chart topics. Although the importance of the consideration of measurement uncertainty was pointed out in many studies, control charts under the presence of measurement error started to get attendance in 2000s. Due to the strong propagation of the importance of measurement uncertainty, higher number of papers with the consideration of measurement errors could be expected in control charts area. This can be explained by the growth of computational power too. Investigation of measurement error effect can be performed through simulations and researchers have been limited by the computational power in the early phase of research regarding control charts under measurement errors.

On the other hand, most of the studies analyzed the effect of the measurement uncertainty but gave no detailed and comprehensive solution or new control chart that is able to reduce the risk of incorrect decisions. However, some papers with the aspect of economic design considered the production costs under the presence of measurement errors, they did not take the risk of decisions like type II. error (prestige loss) into account. Others proposed improved sampling policy to reduce the effect of measurement errors without considering the costs of decision outcomes.

Although, these studies highlighted that measurement uncertainty is important research field in terms of control chart design, there is no proposed method that:

- considers the risk of correct and incorrect decisions about the controlled process
- can be applied under any type of measurement error distribution
- can be used for conformity control or can be extended for control charts
- can be extended for multivariate or adaptive control charts.

Taking the above facts into account, there is a need for a new family of control charts with the combination of the two referred research areas. The newly designed family of control charts should be able to address the aforementioned issues by utilizing the results of both, control chart design and measurement uncertainty / conformity control research areas.

On Figure 2.7, this new direction is illustrated by red dashed lines and in the rest of the dissertation I refer to that as "Risk-based aspect".

2.3 Summary and contribution to literature

In this section, I summarize the most important findings of the systematic literature review with special regard to the deficiencies of the analyzed research areas. Finally, I determine how this research contributes to the scientific literature.

During the literature review, I collected, classified and analyzed the most relevant papers books proceedings and standards regarding control chart design and measurement uncertainty / conformity control research fields. Not only the papers but also citation data were collected in order to refine the search. Networks were built to analyze the structure of both research areas and furthermore, time based introduction was provided to get overview about research trends.

The main findings can be summarized as follows:

1. **Measurement error characteristics:** It was proved that measurement uncertainty should not be treated just as an interval. Several solutions and approaches were proposed to express measurement uncertainty under asymmetric measurement error distribution however, only a few studies considered asymmetric measurement error in conformity control (Figure 2.1). Furthermore, there is no study that analyzes the impact of 3^{rd} and 4^{th} moments of measurement error distribution on the effectiveness of conformity control. Research question Q1 is still valid and further analysis needs to be performed in order to address this issue.

2. **Lack of studies:** Regarding control charts, the most cited and studied fields are traditional univariate, multivariate and adaptive control charts without considering measurement errors during control chart design. Importance of measurement uncertainty was strongly emphasized and in comparison to that, number of papers dealing with control chart under the presence of measurement error is way below the expectations. Measurement uncertainty and its consequence does not get the attendance in control chart design what it deserves based on its importance.

3. **Deficiencies proposed solutions:** The current control chart developments do not address all the issues raised by measurement uncertainty studies (incorrect decisions, prestige loss, asymmetric error distributions). They propose improved sampling strategy or consider production costs only but do not treat the measurement uncertainty as risk factor. Therefore, research question Q1 and Q2 are still valid because I did not find any study that develops a risk-based control chart which was able to overcome the aforementioned problems.

4. **Weak linkage:** The linkage between the two analyzed research areas is weak however, control chart design studies could better rely on results from measurement uncertainty / conformity control area.

5. **Research directions:** Based on the trend analysis, it is clearly visible that control chart researches mainly moved to the direction of adaptive control chart design, nonparametric solutions and risk-based concept did not become significant part of control chart development.

Contribution to the literature

As the outcome of this dissertation, I intend to contribute to the scientific literature in the following way:

1. I investigate the effect of 3^{rd} and 4^{th} moments of measurement error distribution on conformity control strategy and determine which moments need to be considered during the measurement error characterization by conformity or process control.

2. I develop multivariate and adaptive risk-based control charts considering measurement uncertainty (and applying the new knowledge given by point 1). The proposed control charts are able to reduce producer's and consumer's risk as additional contribution compared to the currently used control charts.

3. My research strengthens the linkage between measurement uncertainty and control chart design areas by utilizing both control chart and measurement uncertainty research results in one proposed methodology.

4. Finally, as the main outcome of the dissertation I provide a new family of Risk-based control charts opening a new direction for further researches (Figure 2.7).

In the interest of completeness, I introduce the fitting of my research into the scientific literature in Chapter 7. I provide an additional citation network with the highlighted location of my publications related to the topic of dissertation.

In the next chapter I introduce the proposed methods regarding measurement error characterization and risk-based control chart design.

Chapter 3

Methods

3.1 Characterization of measurement error distribution

In this section, I introduce the structure of the analysis I conduct in order to analyze the effect of measurement error skewness and kurtosis on optimal acceptance strategy. As BIPM et al. (1995) described, the first and second moments (expected value and standard deviation) have significant impact to the distortion effect of measurement uncertainty . Therefore in my thesis, I focus on the effect of third and fourth moments (i.e. skewness and kurtosis) of the measurement error distribution function.

In order to investigate the effect of skewness and kurtosis I use simulation (optimization) procedure as follows:

Let us consider a conformity control process, with the real value of the controlled product characteristic x and the value of the measurement error ε. It is assumed that the probability density functions (pdf) of x and ε are known (Let us note that the pdf of ε can be estimated from the calibrations, or it can be derived from the producer's documentation on the measurement instrument and the measurement system analysis).

The conformity of the product is judged based on the observed (measured) value denoted by y. In the simulation, additive measurement error model is considered as used by Mittag and Stemann (1998):

$$y = x + \varepsilon \tag{3.1}$$

It is necessary to note that characteristics of the measurement error distribution can be obtained in multiple ways:

- Based on experiment: The measurement error distribution parameters (e.g. expected value, standard deviation, etc.) can be estimated through experimental measurements based on a series of independent observations. For detailed guidance, see Eisenhart (1969), Croarkin (1984), NIST (1994), Box et al. (2005), Mandel (2012), Natrella (2013).

- Based on information: This approach is based on other than experimental sources like certified reference materials, calibration reports, industry guides, manufacturer's specifications etc. (Choi et al., 2003a). Further details are provided by NIST (1994).

Characteristics of measurement error distribution can change over time due to different reasons such as aging of the measurement device or environmental causes (vibration, temperature, etc.) Therefore, measurement system needs to be analyzed regularly according to the device's reference manual. An overview of the widely-used measurement system analysis techniques is provided in Appendix F.

3.1.1 Decision outcomes

The product is considered to be conforming if y (observed value) falls between the lower specification limit (LSL) and upper specification limit (USL):

$$LSL \leq y \leq USL \tag{3.2}$$

Nevertheless, the product is conforming only in that case if the real value of the real characteristic falls between these specification limits, i.e.:

$$LSL \leq x \leq USL \tag{3.3}$$

At least four decision outcomes can be distinguished (due to the existence of the measurement error) as a combination of real conformity and decision:

- Correct acceptance
- Correct rejection
- Incorrect acceptance (type II. error)
- Incorrect rejection (type I. error)

Consideration of the several decision outcomes is important, since they might lead to serious consequences from the company's point of view such as increased costs or even prestige loss. Table 3.1 shows the structure of the four decision outcomes.

TABLE 3.1: Cost of decision outcomes as a function of decision and actual conformity

Cost	**Decision**	
	Acceptance (1)	Rejection (0)
Fact		
The product is conforming (1)	c_{11} Correct acceptance	c_{10} Incorrect rejection
The product is non-conforming (0)	c_{01} Incorrect acceptance	c_{00} Correct rejection

Incorrect rejection or type I. error is committed when the observed product characteristic (y) falls outside the acceptance interval, however the product is conformable according to the real value (x):

$$LSL > y \text{ or } y > USL, \text{and } LSL \leq x \leq USL \tag{3.4}$$

Incorrect acceptance is the opposite case (type II. error), when a defected product is accepted due to the distortion of measurement error:

$$LSL > x \text{ or } x > USL, \text{and } LSL \leq y \leq USL \tag{3.5}$$

It is important to notice that the consequences of this error type can be much more serious because purchasing defected products can lead to penalties or even prestige loss for the producer company.

In the remaining two cases, the decisions are correct because the defected product is rejected or the conformable product is accepted.

In Table 3.1, c_{ij} denotes the cost assigned to each decision outcome. Direct production cost (or prime cost) and investigation cost can be counted by all the cases, because manufacturing and investigation are necessary parts of the decision making procedure. It is necessary to note, that in short term, both the measurement and the production process parameters are regarded as constants.

3.1.2 Structure of the simulation

Most of the recommendations assume the normality of measurement error distribution however, by different types of measurement error distributions (normal, triangular, lognormal, gamma, Weibull, binomial, and Poisson), the distribution function can be asymmetric (Herrador and Gonzalez, 2004, D'Agostini, 2004). In that case, expected value and standard deviation are not enough to characterize the distribution function.

Monte-Carlo simulation can be used to obtain information about the relationship between measurement error distribution parameters and optimal acceptance strategy. Monte-Carlo simulation (MCS) is a frequently used approach in the field of optimization, numerical integration and study of probability distributions of random variables (Dyer, 2016, Abonazel, 2018). Its main steps are the followings (Salleh, 2013):

1. Model creation with the appropriate assumptions and input parameters.
2. Random number generation based on step 1.
3. Running the simulation (iteration with modified inputs) and saving of outputs.
4. Analysis of the recorded outputs.

The aforementioned steps are general however, in order to investigate the effect of skewness and kurtosis on the measurement and decision making system, I construct the current simulation including the following steps:

1. Generation of random numbers with normal distribution representing the real values (x) for the measured product characteristic
2. Generation of random numbers (representing the measurement error ε) with Matlab's "*pearsrnd*" function with given skewness and kurtosis
3. Determination of the cost assigned to each decision outcome (c_{ij})
4. Optimization of the acceptance interval in order to minimize the total decision cost
5. Iterate Step 1-4 while changing the skewness and kurtosis of the measurement error distribution

I provide more detailed description about the aforementioned steps of the simulation.

1-2. Generation of process and measurement error First of all x and y values need to be simulated. in this study, I generate the real product characteristic values (x) as random numbers following normal distribution with given expected value and variance. y can be simulated based on Equation (3.1), where ε is generated with Matlab's "*pearsrnd*" function. This function allows the user to generate random numbers with given mean, standard deviation, skewness and kurtosis.

3. Determination of decision costs During the simulation, theoretical cost values are selected for each decision outcome (The applicability will be validated in a practical example as well.). It is important to note that multiple decision cost structures need to be considered like extreme cost for type I. error, extreme cost for type II. error or no extreme cost for any type of incorrect decision. This will make it possible to investigate the behavior of the optimal acceptance interval under different cost structures.

4. Optimization For the modification of acceptance interval, a correction components $K_{LSL}, K_{USL} \in \mathbb{R}$ are applied:

$$LSL_K = LSL + K_{LSL} \text{ and } USL_K = USL - K_{USL} \tag{3.6}$$

where USL$_K$ and LSL$_K$ denote the modified specification limits. Obviously, increase in $|K_{LSL}|$, $|K_{USL}|$ means stricter and decrease of $|K_{LSL}|$, $|K_{USL}|$ means more permissive acceptance policy.

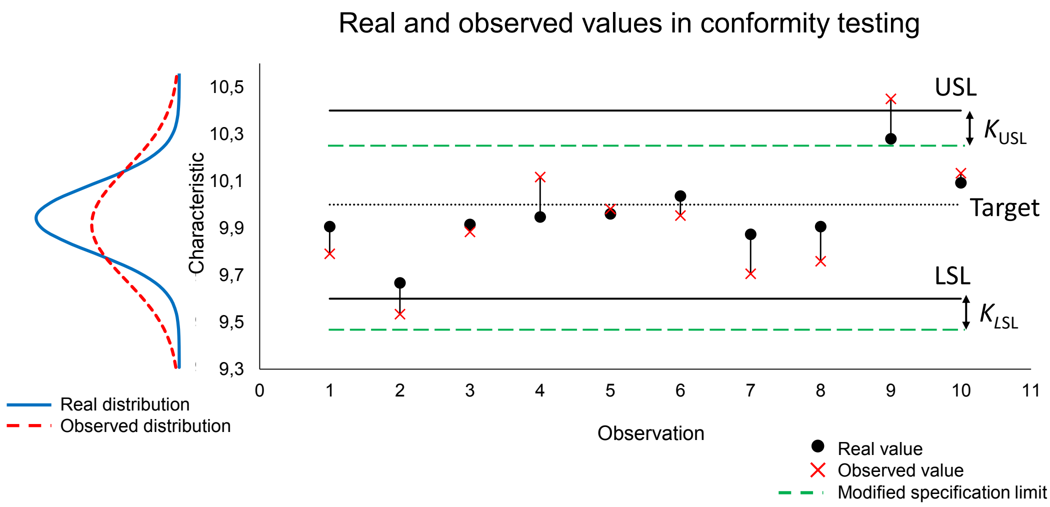

FIGURE 3.1: Illustration of specification limit modification (source: own edition based on Hegedűs (2014))

Total decision cost has to be minimized and objective function can be described as follows:

$$TC = C_{11} + C_{00} + C_{10} + C_{01} = q_{11} \cdot c_{11} + q_{00} \cdot c_{00} + q_{10} \cdot c_{10} + q_{01} \cdot c_{01} \tag{3.7}$$

were TC is the total decision cost, C_{ij} is the aggregated cost of each decision outcome, q_{ij} is the quantity of decisions according to the certain decision outcomes.

In Equation (3.7), the costs of all four decision outcomes appear. Examples for there cost components can be provided as follows:

- Correct acceptance (c_{11}): It includes all the production and inspection costs. Production cost such as material-, labor, operating cost (rent, insurance can be counted as indirect costs). Inspection cost consists of the cost of sampling, labor, operating cost of the measuring equipment.

- Correct rejection (c_{00}): Production and inspection costs also exist in this case however, further factors needs to be taken into account such as cost of re-manufacturing (if possible) or cost of scrap-handling.
- Incorrect rejection (c_{10}): In this case, the cost components are the same as in the correct control however, the consequences are more significant because the manufacturer company needs to consider the fact that it can not sell a product which satisfies the specification. This can be estimated as missed revenue.
- Incorrect acceptance (c_{01}): This case has the most serious consequences. If non-conformable product can be found in the supplied batch, it often means that the manufacturer company has to re-sort the entire batch on its own cost. It also can lead to high penalties according to the contract between producer and customer.

In this aspect, the risk of each decision outcome can be considered as the multiplication of their frequency and the expected cost of the occurrence during the simulation (For further interpretation of risk, see Appendix A.). The goal is to find the optimal values of K_{LSL}, K_{USL} in order to minimize TC.

5. Iteration Skewness and kurtosis of the generated measurement error distribution are changed in every iteration and optimal values of K_{LSL}, K_{USL} are computed. As outcome, the relationship between skewness/kurtosis of measurement error and optimal correction components is analyzed. As it was mentioned before, the simulation is conducted under several decision cost structures and the simulation results are compared.

3.2 Risk-based multivariate control chart

The development of the Risk-based multivariate control chart can be structured as four main steps:

1. Data collection and simulation
2. Construction of the traditional T^2 control chart
3. Specification of decision outcomes and estimation of decision costs
4. Construction of the Risk-based multivariate control chart with the adjustment of control lines

In the next subsections I introduce each step in details.

3.2.1 Data collection and construction of traditional T^2 chart

In order to construct the traditional T^2 chart as first step, it is necessary to collect the required information about the process and measurement error. If the product characteristic and measurement error distributions are known, and their parameters can be estimated, "real" and "detected" product characteristic values can be simulated. I introduce the proposed method assuming a process with two controlled product characteristics (p_1 and p_2).

Let $\boldsymbol{x}_1$ be the vector of the generated real values of product characteristic 1 and $\boldsymbol{x}_2$ the real value-vector of product characteristic 2 respectively. In this case the real product characteristic values follow normal distribution with expected value μ_1 and μ_2 and standard deviation σ_1 and σ_2:

$$\mathbf{x}_1 \sim N_m(\mu_1\mathbf{1}_m, \sigma_1^2\mathbf{I}_m) \textit{ and } \mathbf{x}_2 \sim N_m(\mu_2\mathbf{1}_m, \sigma_2^2\mathbf{I}_m) \tag{3.8}$$

where $\mathbf{1}_m$ denotes the m-dimensional vector in which all the elements are equal to 1 and $\mathbf{I}_m$ denotes the $m \times m$ identity matrix. (Note that the proposed method can be used under non-normality as well, since the optimal control limit will be evaluated through optimization.)

In the same way, measurement error (denoted by ε) is generated assuming normal distribution with expected value 0 and standard deviation σ_ε. The detected value vectors $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are calculated by the sum of $\boldsymbol{x}$ and $\boldsymbol{\varepsilon}$:

$$\boldsymbol{y}_1 = \boldsymbol{x}_1 + \boldsymbol{\varepsilon}_1 \text{ and } \boldsymbol{y}_2 = \boldsymbol{x}_2 + \boldsymbol{\varepsilon}_2 \tag{3.9}$$

If the required process and measurement error distribution parameters are estimated, T^2 chart can be designed for the simulated "real" and "detected" values as described by Subsection 3.2.2.

3.2.2 Construction of traditional T^2 chart

Assume that $\mathbf{X}_i$, $i = 1, 2, 3, ...$ vector represents the p quality characteristics of the monitored product, (the p characteristics can be characterized with p-variate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$). If the process parameters are known, the T^2 statistic follows chi-square distribution:

$$\chi_i^2 = n((\overline{\mathbf{X}}_i) - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}((\overline{\mathbf{X}}_i) - \boldsymbol{\mu}) \tag{3.10}$$

Where n is the sample size, $\overline{\mathbf{X}}_i$ is the sample mean vector of i^{th} subgroup, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ denote the in-control process mean vector and covariance matrix. If mean vector and the covariance matrix for in-control process are unknown, they are determined by the sample mean vector $\overline{\overline{\mathbf{X}}}$ and the sample covariance matrix $\mathbf{S}$:

$$T_i^2 = n((\overline{\mathbf{X}}_i) - \overline{\overline{\mathbf{X}}})'\mathbf{S}^{-1}((\overline{\mathbf{X}}_i) - \overline{\overline{\mathbf{X}}}) \tag{3.11}$$

Where T_i^2 is the value of the T^2 statistic related to i^{th} sample, which follows the transformed form of F-distribution:

$$T^2 \sim \frac{(n-1)p}{n-p} F(p, n-p) \tag{3.12}$$

Control limit (UCL_{T^2}) can be defined as follows (Alt, 1982):

$$UCL_{T^2} = \frac{p(m-1)(n-1)}{m(n-1)-p+1} F(p, m(n-1)-p+1, \lambda) \tag{3.13}$$

Where $n > 1$ is the sample size, m is the size of the population, p is the number of monitored product characteristics, $F(p, m(n-1)-p+1, \lambda)$ denotes F-distribution with p and $m(n-1)-p+1$ degrees of freedom.

The T^2 statistics for "real" values can be calculated with Equation (3.11). Accordingly, the following formula can be used regarding measured/detected values:

$$\widehat{T_i^2} = n(\overline{\mathbf{Y}}_i - \overline{\overline{\mathbf{Y}}})'\mathbf{S}_Y^{-1}\overline{\mathbf{Y}}_i - \overline{\overline{\mathbf{Y}}}) \tag{3.14}$$

Where $\widehat{T_i^2}$ is the value of T^2 statistic, $\overline{\mathbf{Y}}_i$ is the sample mean vector for i^{th} subgroup and $\mathbf{S}_Y$ denotes the covariance matrix according to measured/detected product characteristic values.

According to the described methodology above, T^2 charts need to be designed for both, the real and measured processes. As a next step, decision outcomes can be interpreted by the comparison of the computed T_i^2 and $\widehat{T_i^2}$ values.

3.2.3 Decision outcomes and decision costs

Similarly to Section 3.1, four decision outcomes can be defined as a combination of fact and decision. Nevertheless, there is a significant difference between the decisions in conformity control and process control.

In conformity control, the decisions refer to the acceptance of a product however, in the case of process control the decision applies to the judgment of process in/out-of-control state. When Hotelling's T^2 chart is applied, decision outcomes can be defined as follows:

In-control state:

Correct acceptance:

$$T_i^2 \leq UCL \text{ and } \widehat{T_i^2} \leq UCL \tag{3.15}$$

Incorrect acceptance:

$$T_i^2 \geq UCL \text{ and } \widehat{T_i^2} \leq UCL \tag{3.16}$$

Out of control state:

Correct control:

$$T_i^2 > UCL \text{ and } \widehat{T_i^2} > UCL \tag{3.17}$$

Incorrect control:

$$T_i^2 \leq UCL \text{ and } \widehat{T_i^2} > UCL \tag{3.18}$$

Let a be the decision's result based on the real product characteristic T_i^2, and b the decision's result according to the detected product characteristic $\widehat{T_i^2}$, where:

$$a = \begin{cases} 1 \text{ if } T_i^2 \leq UCL \\ 0 \text{ if } T_i^2 > UCL \end{cases} \tag{3.19}$$

$$b = \begin{cases} 1 \text{ if } \widehat{T_i^2} \leq UCL \\ 0 \text{ if } \widehat{T_i^2} > UCL \end{cases} \tag{3.20}$$

If c_{ab} denotes the cost associated with each decision outcome, the decision costs can be expressed with a decision matrix (Table 3.2).

TABLE 3.2: Decision outcomes when applying multivariate T^2 control chart

	Detected characteristic	
Real characteristic	In control-statement	Out of control statement
In control-state	c_{11}	c_{10}
Out of control state	c_{01}	c_{00}

The c_{ab} proportional costs are used to calculate TC. c_{11} denotes the cost of correct acceptance of the process, while c_{00} is the cost of correct control of the process. c_{10} and c_{01} denote the cost of type I. error and type II. error. Applying these four decision outcomes, TC can be computed with Equation (3.7) (which will be also the objective function).

The cost of each decision outcome can be split into more partitions. In the followings, I give detailed interpretation about the cost structure.

Structure of the decision costs

During the interpretation, it is assumed that the process in-control statement always can be achieved as the result of an intervention (calibration or maintenance of the manufacturing equipment).

Cost of correct acceptance (c_{11}) In case of correct acceptance (correctly detected the in-control statement of the process), the following costs can be interpreted:

- Cost of production (c_p)
- Cost of measurement (c_m)

Production cost arises by all decision outcomes and can be split into proportional (like proportional material cost) and fixed parts (such as cost of lighting in the building). cost of measurement consists of two parts, fixed cost (c_{mf}) and proportional cost (c_{mp}) depending on sample size (n). The fixed measurement cost (e.g., labor, lighting, operational cost of the measurement device) occurs in every measurement irrespective of sample size. In addition, the cost of qualification c_q must be considered (charting, plotting, labor) as well.

Cost of correct control (c_{00}) This cost has two main components: (1) control costs (c_c) and (2) cost of restart after a successful intervention (c_r).

The control cost arises in two phases. The first phase is the analysis of the process shift, where the root cause of the shift and the necessity of maintenance are investigated. The root cause identification is more complex by multivariate case, since it is necessary to identify which product characteristic(s) caused the process shift (Ittzés, 1999). The second phase is the intervention, where the manufacturing machine is stopped and maintenance is conducted. c_r is the cost with regard to the restart of the machine (e.g., energetic cost of heating to operating temperature). (It is necessary to note that production cost and cost of measurement also arises in this case such as by all the decision outcomes.)

Cost of incorrect control (c_{10}) In the case of type I. error, the same cost elements occur like in case of correct intervention. However difference between the two cases comes from over-regulation of the process and unnecessary stoppage, root cause investigation leading to arrears of revenue.

Cost of incorrect acceptance - missed control (c_{01}) Type II. error is occurred, the control chart does not give signal, and the process shift is not detected due to measurement error. Though c_p and c_m also occur here, if the acceptance is incorrect, the proportional value of cost of scraps (c_s) needs to be added as a new element. c_s can be estimated with simulation. If process parameters are known and the process can be modeled the expected quantity of defected products can be estimated from the start of a process shift until in-control state is established again (average time of maintenance can be calculated based on historical data).

3.2.4 Construction of Risk-based T^2 control chart

The third step of the proposed method is the modification of the control limit to minimize TC. Traditional T^2 chart (designed by step 1) is used as initial basis of the RBT^2 chart. By the initial step, UCL_{T^2}=UCL_{RBT^2}, where UCL_{RBT^2} is the upper control limit of the RBT^2 chart (Hotelling's T^2 chart has only upper control limit) and UCL_{T^2} denotes the upper control limit of traditional T^2 chart.

For the modification, a correction component $K \in \mathbb{R}$ is applied. The value of the control limit of the RBT^2 chart can be described as the followings:

$$UCL_{RBT^2} = UCL_{T^2} - K \tag{3.21}$$

During the proposed method, K is optimized using the Nelder-Mead simplex search method, the minimum point of the total cost function (obtained from the simulation) is determined by the algorithm. In the two-dimensional case, the process generates a sequence of triangles and they converge down to the solution point. The advantage of the method, is, that the algorithm can be extended to more function parameters e.g., sampling interval or sample size. The Nelder-Mead method can be applied even in one-dimensional case. In this study, the function variable is the control limit of the chart and the target variable is the total cost of decisions. The algorithm includes the following steps in one-dimensional case:

Let K be the correction component and K' the other initial point in the algorithm. Furthermore $f(K)$ is the cost function.

Ordering:

The first step is the ordering of the points:

$$f(K) \leq f(K') \tag{3.22}$$

There are four operations: reflection, expansion, (inside or outside) contraction and shrink. In each step the point with the highest cost value will be replaced.

Reflection:

The first operation is the reflection where the reflection point (K_r) is determined:

$$K_r = \frac{K+K'}{2} + \alpha(\frac{K+K'}{2} - K') \tag{3.23}$$

Where $\alpha > 0$ is the reflection parameter. The $f(K_r)$ is evaluated and K' is replaced with K_r if $f(K) \leq f(K_r) \leq f(K')$.

Expansion:

After this step, the expansion will be operated if $f(K_r) < f(K')$, where:

$$K_e = \frac{K+K'}{2} + \beta(K_r - \frac{K+K'}{2}) \tag{3.24}$$

Where $\beta > 1$ denotes the expansion parameter. $f(K_e)$ is evaluated according to K_e.If $f(K_e) < f(K_r)$, then K' is replaced with K_e otherwise it is replaced with K_r .

Contraction:

As the next step, outside and inside contraction is operated.

Outside contraction is used if $f(K) \leq f(Kr) < f(K')$:

$$K_{oc} = \frac{K+K'}{2} + \gamma(K_r - \frac{K+K'}{2}) \tag{3.25}$$

Where γ is the contraction parameter ($0 < \gamma < 1$). Then $f(Koc)$ is evaluated and K' is replaced with K_{oc} if $f(K_{oc}) < f(K_r)$.Otherwise go to step 5.

Inside contraction must be used if $f(K_r) \geq f(K')$.

$$K_{ic} = \frac{K+K'}{2} - \gamma(K_r - \frac{K+K'}{2}) \tag{3.26}$$

$f(K_{ic})$ is evaluated and K' is replaced with K_{ic} if $f(K_{ic}) \leq f(K')$. Otherwise go to step 5.

Shrink:

$$K'_2 = K - \delta(K - K') \tag{3.27}$$

where δ is the shrink parameter.

This section introduced the design methodology of the proposed RBT^2 chart. In order to demonstrate the performance of the chart, simulation results will be provided by Section 4.2 and applicability will be verified through practical example in Section 6.2.

3.3 Risk-based adaptive control chart

In this section, I introduce the development of a risk-based adaptive control chart. The steps are the same as in the case of multivariate control chart:

1. Data collection and simulation of the process
2. Construction of traditional VSSI control chart
3. Specification of the decision outcomes and estimation of decision costs
4. Development of risk-based VSSI $\overline{X}$ chart with the adjustment of control lines

3.3.1 Data collection and simulation

As the first step, an $n \times m$ matrix (denoted by $\mathbf{X}$) of the "real" values is generated with expected value μ_x and standard deviation σ_x. Similarly, an $n \times m$ matrix $\mathbf{E}$, representing the measurement error, is also generated. We use Matlab's "*pearsrnd*" function to generate the measurement error matrix. This function returns an $n \times m$ matrix of random numbers according to the distribution in a Pearson system. With this approach, the four parameters (expected value, standard deviation, skewness and kurtosis) of the measurement error distribution can be easily modified.

After these two matrices are generated, the matrix of "observed" values can be estimated in the following manner:

$$\mathbf{Y} = \mathbf{X} + \mathbf{E} \tag{3.28}$$

where $\mathbf{Y}$ is an $n \times m$ matrix containing the estimated observed values.

In both $\mathbf{X}$ and $\mathbf{Y}$, each row represents a possible sampling event and each element in a row represents all the possible products that can be selected for sampling. To construct the VSSI X-bar chart, the VSSI rules must be applied to $\mathbf{X}$ and $\mathbf{Y}$. The algorithm loops through the matrices from the first row to the n^{th} row.

Let $\bar{\mathbf{x}}$ be the vector of sample means from $\mathbf{X}$, and let $\bar{\mathbf{y}}$ be the vector of sample means selected from $\mathbf{Y}$. If the i^{th} sample mean (with sample size n_1) falls within the warning region, n_2 and h_2 must be used in the next sampling:

a, If $\overline{x}_i \in I_2$, then the $i + h_2^{th}$ row from $\mathbf{X}$ is selected for sampling and element n_2 is selected randomly from the $i + h_2^{th}$ row. Otherwise, the $i + h_1^{th}$ row is selected with sample size n_1.

b, If $\overline{y}_i \in I_2$, then the $i + h_2^{th}$ row from $\mathbf{Y}$ is selected for sampling and element n_2 is selected randomly from the $i + h_2^{th}$ row. Otherwise, the $i + h_1^{th}$ row is selected with sample size n_1.

Where I_1 denotes the central region, and I_2 the warning region that can be described as follows (Chen et al., 2007):

$$I_1(i) = \left[\frac{\mu_0 - w\sigma}{\sqrt{n(i)}}, \frac{\mu_0 + w\sigma}{\sqrt{n(i)}} \right] \tag{3.29}$$

and

$$I_2(i) = \left[\frac{\mu_0 - k\sigma}{\sqrt{n(i)}}, \frac{\mu_0 - w\sigma}{\sqrt{n(i)}} \right] \cup \left[\frac{\mu_0 + w\sigma}{\sqrt{n(i)}}, \frac{\mu_0 + k\sigma}{\sqrt{n(i)}} \right] \tag{3.30}$$

$$I_3(i) = \overline{I_1 \cup I_2} \tag{3.31}$$

where $i = 1, 2...$ is the number of the sample, (n_1, h_1) is the first level of parameter set including smaller sample size (n_1) and longer sampling interval (h_1) and (n_2, h_2) is a second level parameter set with a larger sample size (n_2) and shorter sampling interval (h_2) ($n_1 < n_0 < n_2$ and $h_2 < h_0 < h_1$, where n_0 is the sample size and h_0 is the sampling interval of the FP control chart). Detailed description about VSSI rules is provided in Appendix C.

3.3.2 Construction of traditional VSSI $\overline{X}$ chart

As a next step, the "traditional" VSSI $\overline{X}$ chart (In this context, "traditional" means that the chart does not consider the effect of measurement errors.) can be designed using simulated data from step 1. Upper and lower warning limits (UWL and LWL) can be calculated based on Equation (3.29) and similarly, Equation (3.30) can be used to compute the control limits of the control chart (UCL, LCL). Then, switching rules between (n_1, h_1) and (n_2, h_2) can be used based on the consideration of warning- and control lines.

As output of step 2, "traditional" VSSI $\overline{X}$ charts are designed for real and measured values as well. Decision outcomes can be interpreted by comparing the location of $\overline{x}_i$ and $\overline{y}_i$ sample means related to the warning- and control limits.

3.3.3 Decision outcomes and decision costs

Four type of decision outcomes could be defined in case of conformity control and T^2 chart (See Table 3.2.3). When using adaptive control chart, the number of decision outcomes can be extended due to the existence of warning limits that brings additional aspects to the structure of decision outcomes as it is shown by Table 3.3:

TABLE 3.3: Decision outcomes when using VSSI $\overline{X}$ chart

<table>
<tr><td colspan="3" rowspan="3"></td><td colspan="4">Detected product characteristic</td></tr>
<tr><td colspan="2">in (CL)</td><td colspan="2">out (CL)</td></tr>
<tr><td>in (WL)</td><td>out (WL)</td><td>in (WL)</td><td>out (WL)</td></tr>
<tr><td rowspan="4">Real</td><td rowspan="2">in (CL)</td><td>in (WL)</td><td>$\overline{x}_i \in I_1$ and $\overline{y}_i \in I_1$ (1)</td><td>$\overline{x}_i \in I_1$ and $\overline{y}_i \in I_2$ (2)</td><td></td><td>$\overline{x}_i \in I_1$ and $\overline{y}_i \in I_3$ (3)</td></tr>
<tr><td>out (WL)</td><td>$\overline{x}_i \in I_2$ and $\overline{y}_i \in I_1$ (4)</td><td>$\overline{x}_i \in I_2$ and $\overline{y}_i \in I_2$ (5)</td><td></td><td>$\overline{x}_i \in I_2$ and $\overline{y}_i \in I_3$ (6)</td></tr>
<tr><td rowspan="2">out (CL)</td><td>in (WL)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>out (WL)</td><td>$\overline{x}_i \in I_3$ and $\overline{y}_i \in I_1$ (7)</td><td>$\overline{x}_i \in I_3$ and $\overline{y}_i \in I_1$ (8)</td><td></td><td>$\overline{x}_i \in I_3$ and $\overline{y}_i \in I_1$ (9)</td></tr>
</table>

In Table 3.3, terms "in(CL)" and "out(CL)" denote the in-control and out-of-control statements based on the control line(s), and "in(WL)" and "out(WL)" represent the sample location relative to the warning limits. In addition, $\overline{x}_i$ is the real sample mean, and $\overline{y}_i$ is the detected (measured) mean related to the i^{th} sampling. I_1, I_2 and I_3 denote the regions based on Equations (3.29), (3.30) and (3.31). Some combination cannot be interpreted i.e. if $\overline{y}_i$ falls within the out-of-control region based on the control line, it excludes the potential of being "in-control" based on the warning limit. Similarly, if $\overline{x}_i$ > UCL or $\overline{x}_i$ < LCL (process if out-of control based on the control lines), then LWL < $\overline{x}_i$ < UWL statement cannot be true. These cases are represented by gray-colored cells in the table. Detailed explanation of each (9) case is also provided in the following:

- Case 1: Both the detected and the real sample mean fall within the central region. The decision is a correct acceptance.
- Case 2: The detected sample mean is in the warning region but the real sample mean is in the central region. In this case, the sample size is increased and the sampling interval is reduced. However, these changes are unnecessary, and the decision is incorrect.
- Case 3: The process is out-of-control based on $\overline{y}_i$, but $\overline{x}_i$ falls within the central region. The expected value of the process is in-control, but a shift is detected incorrectly. Therefore, an unnecessary corrective action is taken (type I. error).
- Case 4: $\overline{x}_i$ is within warning region (out-of-control based on the warning limit) but an in-control statement is detected. In this case, the sample size should be increased and the sampling interval should be reduced; however, this action is not taken. This failure reduces the performance of the control chart because it delays the time for detection and correction.
- Case 5: Both the detected and real sample mean fall within the warning region. Sample size is increased, sampling interval is reduced as part of a correct decision.
- Case 6: Out-of-control state is detected; however, the $\overline{x}_i$ falls within the warning region. Corrective action is taken, but switch between the chart parameter sets (n, h) would be enough. The decision is incorrect.
- Case 7: In-control state is detected and $\overline{y}_i$ is located in the central region however, the process is out-of-control. The decision is incorrect, and corrective action is not taken (type II. error).
- Case 8: Similar to Case 7, but $\overline{y}_i$ is in the warning region. Therefore, this case is more advantageous compared to Case 7 because a strict control policy is applied and therefore, shorter time is needed to detect process shift.
- Case 9: The process is out-of-control based on real and detected sample means; therefore, the decision is correct.

For better clarification, Figure 3.2 illustrates the nine decision outcomes described above.

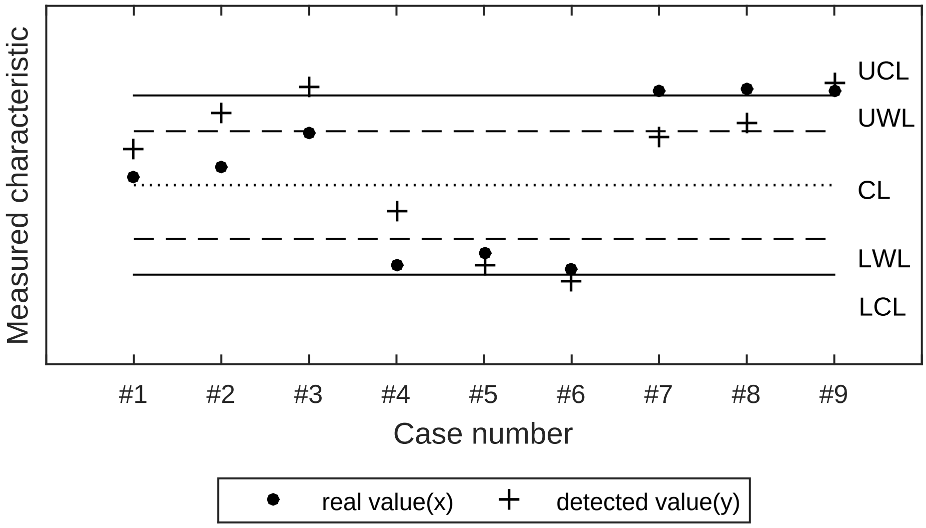

FIGURE 3.2: Demonstration of the nine decision outcomes on a control chart

Structure of the decision costs

In the followings I introduce the cost structure of each decision outcome. Each decision cost consists of several elements therefore, I collected those parameters that are used during the specification of the decision costs (Table 3.4).

TABLE 3.4: Elements of the cost of decision outcomes

Symbol	Name
n	sample size
N_h	produced quantity in the considered interval (h)
c_p	production cost
c_{mf}	fixed cost of measurement
c_{mp}	proportional cost of measurement
c_q	cost of qualification
c_s	cost of switching
d_1	weight parameter for switching
c_i	cost of intervention
d_2	weight parameter for intervention
c_{rc}	cost of root cause search
c_{id}	cost of delayed intervention
c_f	cost of false alarm identification
c_{mi}	cost of missed intervention
c_r	cost of restart
c_{ma}	maintenance cost

Table 3.4 shows the specified cost components in the cost structure. The following costs are involved in each decision:

- expected total production cost
- cost of measuring

- cost of qualification

c_p denotes the proportional production cost and N_h is the expected number of manufactured products in h (where h denotes the time interval between two samples). Therefore, the expected total production cost can be estimated as $N_h c_p$. The cost of measurement consists of two parts, fixed cost (c_{mf}) and proportional cost (c_{mp}) depending on sample size (n). The fixed measurement cost (e.g., labor, lighting, operational cost of the measurement device) occurs in every measurement irrespective of n. c_{mp} is the expected measurement cost related to a sample that strongly depends on sample size (this is especially significant for destructive measurement processes). Thus, the expected total measurement cost can be estimated as:

$$\text{total measurement cost} = nc_{mp} + c_{mf} \tag{3.32}$$

In addition, the cost of qualification c_q must be considered (charting, plotting, labor) as well. Accordingly, since $N_h c_p + nc_{mp} + c_{mf} + c_q$ is part of each cost component, a c_0 constant is applied as simplification:

$$N_h c_p + nc_{mp} + c_{mf} + c_q = c_0 \tag{3.33}$$

Some cost components occur in special cases only. The cost of switching c_s is the cost associated with modification of the VSSI chart parameters (n, h). c_i denotes the cost of intervention, including the cost of stoppage and root cause search (c_{rc}). If the root cause cannot be identified, it means that probably false alarm occurred. In this case, there is no maintenance cost (c_{ma}) however, cost of false alarm identification (c_f) needs to be considered. On the other hand, when a root cause is found, the machine must be maintained (e.g., cost of the replaced parts, labor cost).

The weighting parameters (d_1, d_2) must also be specified. Some cases (e.g. Case 2 and Case 5) are similar but have different estimated costs. This difference comes from the necessity of the decision. For example, in Cases 2 and 5, $\overline{y}_i$ is located in the warning region but the parameter switch (n_1 to n_2 and n_1 to h_2) is necessary in Case 2 and unnecessary in Case 5. In similar cases, the unnecessary decision must be multiplied by the weighting parameter in order to penalize surplus modifications during control. Therefore, d_1 is the weighting parameter for the cost of unnecessary switching, and d_2 is the weighting parameter for unnecessary intervention. Table 3.4 includes the forms of the decision costs assigned to the decision outcomes.

TABLE 3.5: Structure of the decision costs (VSSI control chart)

Case	Structure	Simplified form
#1	$C_1 = N_h c_p + nc_{mp} + c_{mf} + c_q$	$C_1 = c_0$
#2	$C_2 = N_h c_p + nc_{mp} + c_{mf} + c_q + d_1 c_s$	$C_2 = c_0 + d_1 c_s$
#3	$C_3 = N_h c_p + nc_{mp} + c_{mf} + c_q + d_2 c_i$	$C_3 = c_0 + d_2 c_i$
#4	$C_4 = N_h c_p + nc_{mp} + c_{mf} + c_q + c_{id}$	$C_4 = c_0 + c_{id}$
#5	$C_5 = N_h c_p + nc_{mp} + c_{mf} + c_q + c_s$	$C_5 = c_0 + c_s$
#6	$C_6 = N_h c_p + nc_{mp} + c_{mf} + c_q + d_2 c_i$	$C_6 = c_0 + d_2 c_i$
#7	$C_7 = N_h c_p + nc_{mp} + c_{mf} + c_q + c_{mi}$	$C_7 = c_0 + c_{mi}$
#8	$C_8 = N_h c_p + nc_{mp} + c_{mf} + c_q + d_3 c_{mi}$	$C_8 = c_0 + c_{mi}$
#9	$C_9 = N_h c_p + nc_{mp} + c_{mf} + c_q + c_{ma} + c_r$	$C_9 = c_0 + c_r$

During the control process, the appropriate decision cost must be assigned to each sampling point. The assigned costs must be further aggregated to determine the total decision cost:

$$TC = q_1c_0 + q_2(c_0 + d_1c_s) + q_3(c_0 + d_2c_i) + \\ + q_4(c_0 + c_{id}) + q_5(c_0 + c_s) + q_6(c_0 + d_2c_i) + \\ + q_7(c_0 + c_{mi}) + q_8(c_0 + d_3c_{mi}) + q_9(c_0 + c_{ma} + c_r) \tag{3.34}$$

Or a simplified form can be used as well:

$$TC = \sum_{i=1}^{9} q_iC_i \tag{3.35}$$

where $i = 1, 2, ..., 9$ is the case number, q_i denotes the quantity of decision points (or samples) and C_i is the cost related to the i^{th} case. The goal is to find the optimal value of the coverage factor k (and the optimal values of the control lines UCL, LCL) and the optimal parameter set for the switching (n, h) to minimize TC.

3.3.4 Construction of the RB VSSI $\overline{X}$ chart

In order to minimize the objective function described by Equation (3.34) or (3.35), coverage factors k, w and variable parameters (n_1, n_2, h_1, h_2) are optimized. Two approaches are used to optimize the control chart parameters. The integer parameters, (n_1, n_2, h_1, h_2) are optimized using genetic algorithms as the first step (The previously used Nelder-Mead algorithm cannot handle integer parameters.). In the second step, the Nelder-Mead algorithm is used as a hybrid function to optimize the continuous parameters (k, w) and obtain more precise results.

Simulation of the control procedure and optimization

The aforementioned Genetic Algorithm (GA) imitates the principles of natural selection and can be applied to estimate the optimal design parameters for statistical control charts. In the first step, this method generates an initial set of feasible solutions and evaluates them using a fitness function. In the next step, the algorithm:

1. selects parents from the population
2. creates crossover from the parents
3. performs mutation on the population given by the crossover operator
4. evaluates the fitness value of the population.

The steps are repeated until the algorithm finds the best fitting solution (Chen et al., 2007). Then, the Nelder-Mead method is applied as a hybrid function to find the optimal values w and k.

This is a two-dimensional case, where the Nelder-Mead algorithm generates sequence of triangles converging to the optimal solution. The objective function can be described as $C(n_1, n_2, h_1, h_2, w, k)$, where w is the warning limit coefficient and k is the control limit coefficient. Note that the integer parameters (n_1, n_2, h_1, h_2) were already optimized by genetic algorithms; therefore, by this step, $C(w, k)$ can be used as objective function with the following constrains: $0 < w < k$ and $w, k \in \mathbb{R}$.

In two-dimensional case, three vertices are determined and the cost function is evaluated for each vertex. In the first step, ordering is performed on the vertices:

Ordering:

The vertices must be ordered based on the evaluated values of the cost function:

$$C_B(w_1, k_1) < C_G(w_2, k_2) < C_W(w_3, k_3) \tag{3.36}$$

where C_B is the best vertex with the lowest total cost, C_G (good) is the second-best solution and C_W is the worst solution (with the highest cost value). Furthermore, let $\mathbf{v}_1 = (w_1, k_1)$, $\mathbf{v}_2 = (w_2, k_2)$ and $\mathbf{v}_3 = (w_3, k_3)$ represent the vectors of each point.

The approach applies four operations: reflection, expansion, contraction and shrinking (same steps as it was described in Sewct

Reflection:

The reflection point is calculated as:

$$\begin{aligned}\mathbf{v}_R &= [w_R, k_R]^T \\ &= \left[\frac{w_1 + w_2}{2} + \alpha\left(\frac{w_1 + w_2}{2} - w_3\right), \frac{k_1 + k_2}{2} + \alpha\left(\frac{k_1 + k_2}{2} - k_3\right)\right]^T \\ &= \frac{\mathbf{v}_1 + \mathbf{v}_2}{2} + \alpha\left(\frac{\mathbf{v}_1 + \mathbf{v}_2}{2} - \mathbf{v}_3\right)\end{aligned} \tag{3.37}$$

where $\mathbf{v}_{2_S}$ and $\mathbf{v}_{3_S}$ are the shrunk points derived from $\mathbf{v}_2$ and $\mathbf{v}_3$, respectively (Fan et al., 2006).

Expansion:

After reflection, expansion is performed if $C_R(w_R, k_R) < C_B(w_B, k_B)$ condition is true:

$$\begin{aligned}\mathbf{v}_E &= [w_E, k_E]^T \\ &= \left[\frac{w_1 + w_2}{2} + \beta\left(w_R - \frac{w_1 + w_2}{2}\right), \frac{k_1 + k_2}{2} + \beta\left(k_R - \frac{k_1 + k_2}{2}\right)\right]^T \\ &= \frac{\mathbf{v}_1 + \mathbf{v}_2}{2} + \beta\left(\mathbf{v}_R - \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}\right)\end{aligned} \tag{3.38}$$

where $\mathbf{v}_E$ denotes the reflection point with coordinates w_E and k_E and β is the expansion parameter. $C_E(w_E, k_E)$ is evaluated and $\mathbf{v}_3$ is replaced with $\mathbf{v}_E$ if $C_E(w_E, k_E) \leq C_R(w_R, k_R)$.

Contraction:

Outside contraction is performed if $C_G(w_2, k_2) \leq C_R(w_R, k_R) < C_W(w_3, k_3)$:

$$\begin{aligned}\mathbf{v}_{OC} &= [w_{OC}, k_{OC}]^T \\ &= \left[\frac{w_1 + w_2}{2} + \gamma\left(w_R - \frac{w_1 + w_2}{2}\right), \frac{k_1 + k_2}{2} + \gamma\left(k_R - \frac{k_1 + k_2}{2}\right)\right]^T \\ &= \frac{\mathbf{v}_1 + \mathbf{v}_2}{2} + \gamma\left(\mathbf{v}_R - \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}\right)\end{aligned} \tag{3.39}$$

Where $\mathbf{v}_{OC}$ is the point given by outside contraction with coordinates w_{OC} and k_{OC}; furthermore, $0 < \gamma < 1$ is the contraction parameter. Then, $C_{OC}(w_{OC}, k_{OC})$ is evaluated. If $C_{OC}(w_{OC}, k_{OC}) \leq C_R(w_R, k_R)$, replace $\mathbf{v}_3$ with $\mathbf{v}_{OC}$; otherwise, shrinking operation is performed.

The inside contraction point denoted by $\mathbf{v}_{IC}$ is computed if $C_R(w_R, k_R) \geq C_W(w_3, k_3)$:

$$\begin{aligned}\mathbf{v}_{IC} &= [w_{IC}, k_{IC}]^T \\ &= \left[\frac{w_1 + w_2}{2} - \gamma\left(w_R - \frac{w_1 + w_2}{2}\right), \frac{k_1 + k_2}{2} - \gamma\left(k_R - \frac{k_1 + k_2}{2}\right)\right]^T \\ &= \frac{\mathbf{v}_1 + \mathbf{v}_2}{2} - \gamma\left(\mathbf{v}_R - \frac{\mathbf{v}_1 + \mathbf{v}_2}{2}\right)\end{aligned} \quad (3.40)$$

In this case, $C_{IC}(w_{IC}, k_{IC})$ is evaluated, and the point with the highest total decision cost $\mathbf{v}_3$ is replaced with $\mathbf{v}_{IC}$; otherwise, shrinking operation is performed.

Shrinking:

Shrinking must be performed for the n^{th} and $n + 1^{th}$ points. In two-dimensional case (we have parameters w and k), this operation is performed for $\mathbf{v}_2$ and $\mathbf{v}_3$:

$$\mathbf{v}_{2_S} = [w_{2_S}, k_{2_S}]^T = [w_1 + \delta(w_2 - w_1), k_1 + \delta(k_2 - k_1)]^T = \mathbf{v}_1 + \delta(\mathbf{v}_2 - \mathbf{v}_1) \quad (3.41)$$

$$\mathbf{v}_{3_S} = [w_{3_S}, k_{3_S}]^T = [w_1 + \delta(w_3 - w_1), k_1 + \delta(k_3 - k_1)]^T = \mathbf{v}_1 + \delta(\mathbf{v}_3 - \mathbf{v}_1) \quad (3.42)$$

where $\mathbf{v}_{2_S}$ and $\mathbf{v}_{3_S}$ are the shrunk points derived from $\mathbf{v}_2$ and $\mathbf{v}_3$, respectively (Fan et al., 2006).

With the two aforementioned algorithms, both integer and continuous parameters can be optimized and the total decision cost can be reduced with the consideration of measurement uncertainty. The designed new RB VSSI $\overline{X}$ chart has modified warning and control lines compared to the initial adaptive control chart that was designed in step 2.

In order to analyze the performance and verify the applicability of the proposed RB VSSI $\overline{X}$ chart, simulation results are introduced by Section 4.3 and verification through practical example by Section 6.3.

Chapter 4

Simulation results

In this chapter I follow the three-fold structure according to each research proposal. The next section shows the simulation results related to the effect 3^{rd} and 4^{th} moments of the measurement error distribution function on conformity control.

4.1 Characterization of measurement error distribution

In this simulation I assume that the characteristics of measurement error distribution are known. Different levels with regard to skewness and kurtosis of measurement error distribution are simulated. Measurement error is generated with the "pearsrnd" Matlab function, which returns a vector of random numbers derived from a distribution of Pearson system with specified moments (mean, standard deviation, skewness, kurtosis). In the simulation, mean and standard deviation of the measurement error are constant, and only skewness and kurtosis are modified in each iteration. Impact of skewness/kurtosis related to the optimal specification interval is investigated considering:

1. total inspection
2. acceptance sampling

Furthermore, analysis of the effect of skewness and kurtosis, is provided taking three different cost structures into account:

1. Extreme cost regarding type II. error
2. Extreme cost regarding type I. error
3. No extreme cost for any decision outcome

The assumed real values of the product characteristic (x) are normally distributed with expected value 10 and standard deviation 0.2. The product has only a lower specification limit LSL=9.7; furthermore, the expected value of the measurement error (ε) is 0, and $\sigma_\varepsilon = 0.02$ (standard deviation of the measurement error). The optimal correction component K^* is evaluated for each skewness/kurtosis combination (Since there is only one specification limit, I use K^* to denote the optimal correction component for the simplicity.). Table 4.1 shows the cost structures and the maximum, minimum and mean values of K^* correction component results.

TABLE 4.1: Cost structure and result of the simulation

	c_{11}	c_{01}	c_{10}	c_{00}	K^*_{mean}	K^*_{max}	K^*_{min}
Skewness	10	2000	100	50	0.0318	0.042	0.019
	10	200	1000	50	-0.025	-0.018	-0.036
	10	200	100	50	0.003	0.013	-0.007
Kurtosis	10	2000	100	50	0.031	0.038	0.023
	10	200	1000	50	-0.024	-0.019	-0.030
	10	200	100	50	0.003	0.007	-0.001

Figure 4.1 shows the relationship between skewness/kurtosis of measurement error and K^*. The first row includes the results related to the change of skewness and second row shows K^* values as a function of kurtosis respectively. Linear fitting was performed, and R^2 values were also provided to analyze the goodness of fit. As it was mentioned above, the simulation was conducted under different cost structures represented by each column on Figure 4.1.

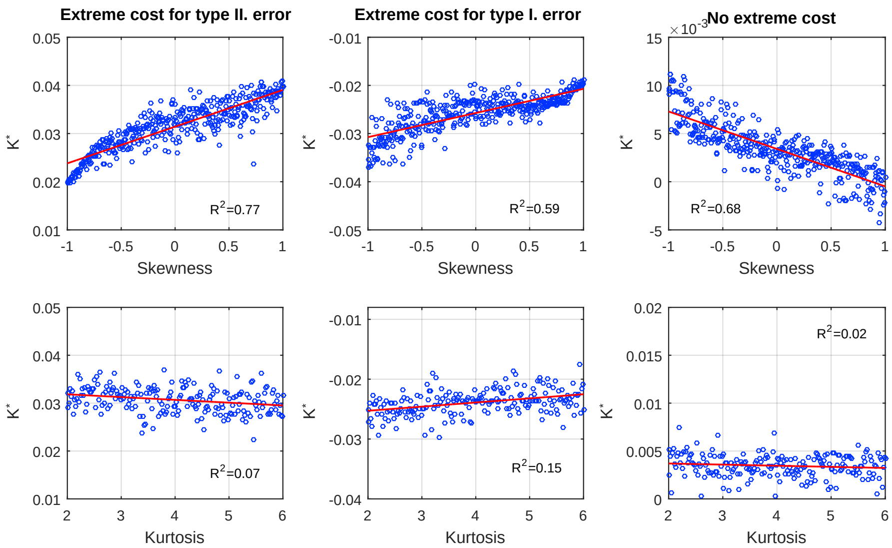

FIGURE 4.1: Optimal values of the correction component (K^*) as a function of skewness and kurtosis of the measurement error distribution (total inspection)

The control policy becomes even stricter when the skewness approaches 1 (note that higher K^* means narrower acceptance interval, since $LSL_K = LSL + K$). Strong relationship between the analyzed variables is also confirmed by the value of R^2 (R^2 = 0.77). Due to the extreme type II. error cost, the algorithm applies strict control policy, even though it increases the probability of type I. error.

In the opposite case, acceptance interval expands while skewness approaches 1. Since the commitment of type I. error causes extreme cost, the absolute value of K^* decreases to minimize the total decision cost through the reduction of the amount of type I. errors.

A trend with negative gradient can also be observed when no extreme costs are assumed. More permissive control policy is applied when the skewness is approaching 1. Since neither type II. error nor type I. error has extreme cost, the algorithm tries to reduce the number of non-conformable products in order to decrease the total decision cost.

An important observation from Figure 4.1 can be stated while looking at the second row of charts. The relationship between K^* and kurtosis of the measurement error density function is inconsiderable in all three cases (the slopes of the fitted lines are nearly zero, and R^2 values are very low).

The results show that not only first and second moments needs to be considered by the characterization of measurement error, but third moment has considerable impact to the effectiveness of the total inspection procedure, while kurtosis of the measurement error has no significant influence.

In the next part of the analysis, the same simulation was conducted, but acceptance sampling was assumed now. The lot size is N = 150000, the sample size is M = 800 and the acceptable number of nonconforming d = 12 which is chosen based on AQL = 1% value from Table 2-B of ISO 2859-1:1999 recommendation single sampling plans for tightened inspection. Same cost structure is assumed as in the case of total inspection (no extreme cost for any decision outcome). Figure 4.2 shows the results of the simulation.

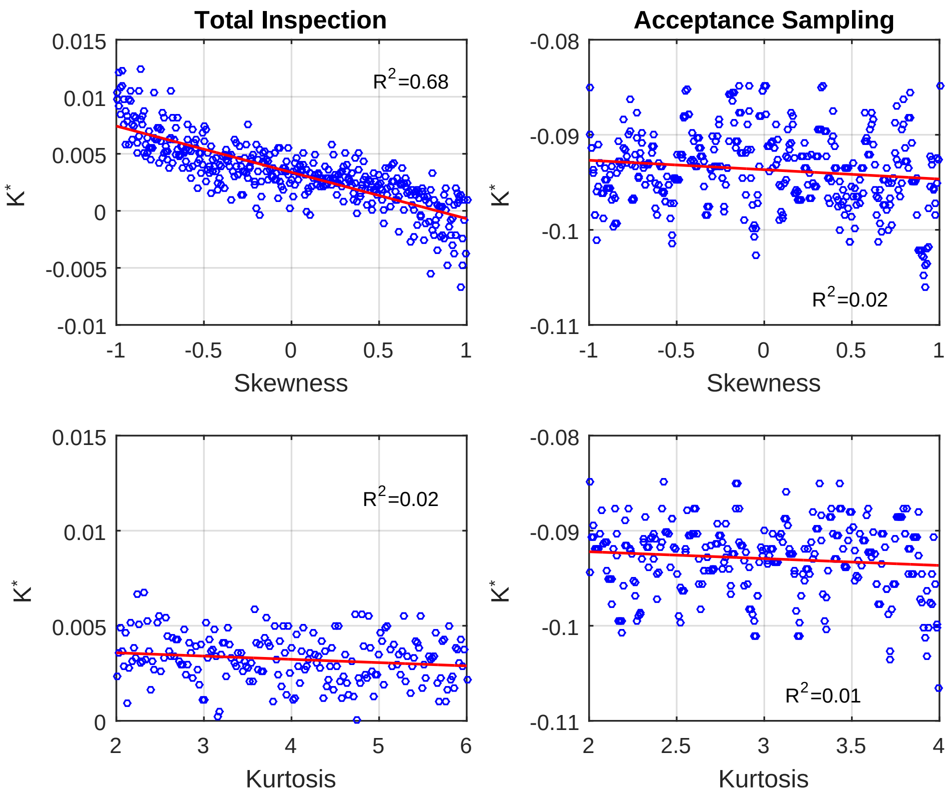

FIGURE 4.2: Optimal values of the correction component (K^*) as a function of skewness and kurtosis of the measurement error distribution (acceptance sampling)

The left charts of Figure 4.2 show the result of the total inspection simulation as reference (from Figure 4.1), while the results according to acceptance sampling can be observed at the right-hand side.

It is clearly visible that neither skewness nor kurtosis has effect on K^* when acceptance sampling is applied (confirmed by the R^2 values as well) because the uncertainty from sampling conceals the measurement uncertainty (central limit theorem).

Conclusion of the analysis:

In the case of total inspection, 3^{rd} moment of measurement error distribution needs to be considered by the characterization of measurement uncertainty but moment 4 can be disregarded. If acceptance sampling is applied, neither skewness nor 4^{th} kurtosis of the measurement error distribution can be used to characterize measurement uncertainty.

4.2 Risk-based multivariate control chart

Consider a product having two main product characteristics (denoted by p_1 and p_2) that need to be controlled simultaneously. Both product parameters follow normal distribution with expected value μ_1, μ_2 and standard deviation σ_1 and σ_2 respectively. The real values of product parameters p_1 and p_2 are denoted by $\mathbf{x_1}$ and $\mathbf{x_2}$ vectors.

As measurement error, random numbers were generated from normal distribution with expected value $\mu_\varepsilon = 0$ and standard deviation $\sigma_\varepsilon = 0.012$ (The measurement error vectors are denoted by $\boldsymbol{\varepsilon_1}$ and $\boldsymbol{\varepsilon_2}$.). In addition, it is also assumed that the two product parameters are measured with the same device and therefore, the measurement error distribution has the same characteristics regardless of which product parameter is measured.

1,000,000 sampling events are simulated with sample size n=1. The permitted false alarm rate (λ) is 0.01. Table 4.2 summarizes the input parameters of the simulation.

TABLE 4.2: Input parameters of the simulation

Input parameters	**Symbol**	**Value**
Number of controlled product characteristics	p	2
Expected value of product parameter 1	μ_1	25.6
Expected value of product parameter 2	μ_2	10.2
Standard deviation of product parameter 1	σ_1	0.07
Standard deviation of product parameter 1	σ_2	0.10
Standard deviation of the measurement error	σ_ε	0.012
Cost related to correct accepting	c_{11}	1
Cost related to correct rejecting	c_{00}	5
Cost related to incorrect accepting	c_{01}	60
Cost related to incorrect rejecting	c_{10}	5
Number of sampling	m	10^6
Sample size	n	1
Permitted false alarm rate to the T^2 chart	λ	0.01

The simulation was conducted considering two aspects. In the first case, the knowledge of the real product characteristics ($\boldsymbol{x_1}$ and $\boldsymbol{x_2}$) is assumed, and the detected values ($\boldsymbol{y_1}$ and $\boldsymbol{y_2}$) are evaluated using Equation (3.9).

In the second case, I assume that only detected product characteristics ($\boldsymbol{y_1}$, $\boldsymbol{y_2}$) can be obtained and real values ($\boldsymbol{x_1}$, $\boldsymbol{x_2}$) are estimated by the difference of obverved value and the estimated measurement error:

$$x_1 = y_1 - \varepsilon_1 \text{ and } x_2 = y_2 - \varepsilon_2 \tag{4.1}$$

T_i^2 and also $\widehat{T_i^2}$ values were calculated to estimate decision costs and compare the performance of the proposed method with Hotelling's T^2 chart. Figure 4.3A and Figure 4.3B show the convergence to the optimum solution during the optimization and Table 4.3 summarizes the simulation results.

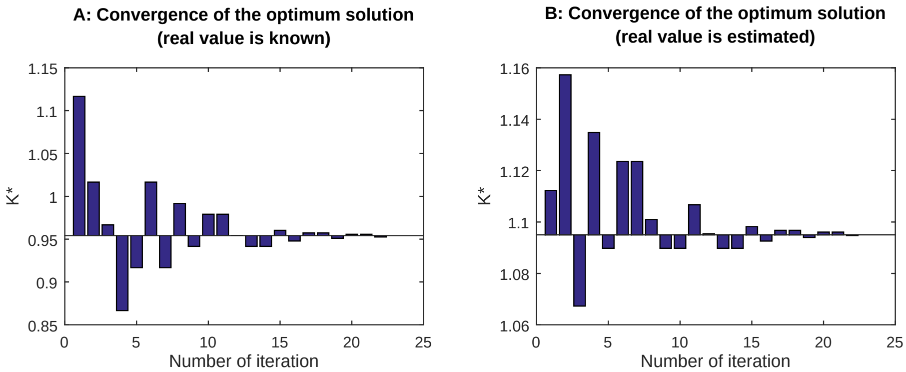

FIGURE 4.3: Convergence to the optimum solution

TABLE 4.3: Performance of RBT^2 chart

Control chart	q_{11}	q_{01}	q_{10}	q_{00}	K	UCL	TC	$\Delta C\%$
T^2	98,851	159	175	815	0.00	9.21	113,341	-
RBT^2 (real value is known)	98,371	28	655	946	0.96	8.25	108,056	4.89
RBT^2 (real value is estimated)	98,229	31	753	987	1.10	8.10	108,789	4.18

When real product characteristic values were known, TC could be decreased by 4.89% with control line adjustment. However, the results show that RBT^2 chart is able to reduce the total decision cost regardless of real value is known or it was estimated. The proposed method tries to reduce the overall decision cost even though it causes additional type I. errors (Note that occurrence of type II. error has more serious consequences associating with higher costs/losses). 159 missed control occurred while using Hotelling's T^2 chart and only 31 in the case of the RBT^2 chart (if the real product characteristics are estimated).

Conclusion of the analysis:

Consideration of measurement uncertainty can reduce the decision cost in the case of multivariate control chart. In the provided simulation, RBT^2 chart was able to reduce the decision costs by nearly 5%.

4.3 Risk-based adaptive control chart

In this section, I demonstrate the performance of the proposed RB VSSI $\overline{X}$ chart through simulations. First the decision costs must be specified (Table 4.4):

TABLE 4.4: Cost values during the simulation (RB VSSI $\overline{X}$ chart)

Case	Structure	Value
#1	$C_1 = N_h c_p + n c_{mp} + c_{mf} + c_q$	1
#2	$C_2 = N_h c_p + n c_{mp} + c_{mf} + c_q + d_1 c_s$	5
#3	$C_3 = N_h c_p + n c_{mp} + c_{mf} + c_q + d_2 c_i$	50
#4	$C_4 = N_h c_p + n c_{mp} + c_{mf} + c_q + c_{id}$	7
#5	$C_5 = N_h c_p + n c_{mp} + c_{mf} + c_q + c_s$	3
#6	$C_6 = N_h c_p + n c_{mp} + c_{mf} + c_q + d_2 c_i$	50
#7	$C_7 = N_h c_p + n c_{mp} + c_{mf} + c_q + c_{mi}$	600
#8	$C_8 = N_h c_p + n c_{mp} + c_{mf} + c_q + d_3 c_{mi}$	550
#9	$C_9 = N_h c_p + n c_{mp} + c_{mf} + c_q + c_{ma} + c_r$	20

The simulated production process follows normal distribution with expected value $\mu_x = 100$ and standard deviation $\sigma_x = 0.2$. Measurement errors follow also normal distribution with expected value $\mu_\varepsilon = 0$ and standard deviation $\sigma_\varepsilon = 0.02$. In the first step, $k = 3$ and $w = 2$ are used to calculate the control and warning limits. The integer design parameters (n_1, n_2, h_1, h_2) are optimized as well to minimize the total cost of the decisions, as described by Equations (3.34) and (3.35). As next step, parameters k and w are optimized using the Nelder-Mead direct search method (k^* and w^* denote the optimal values of k and w). Figure 4.4 shows the convergence of objective function value during the optimization.

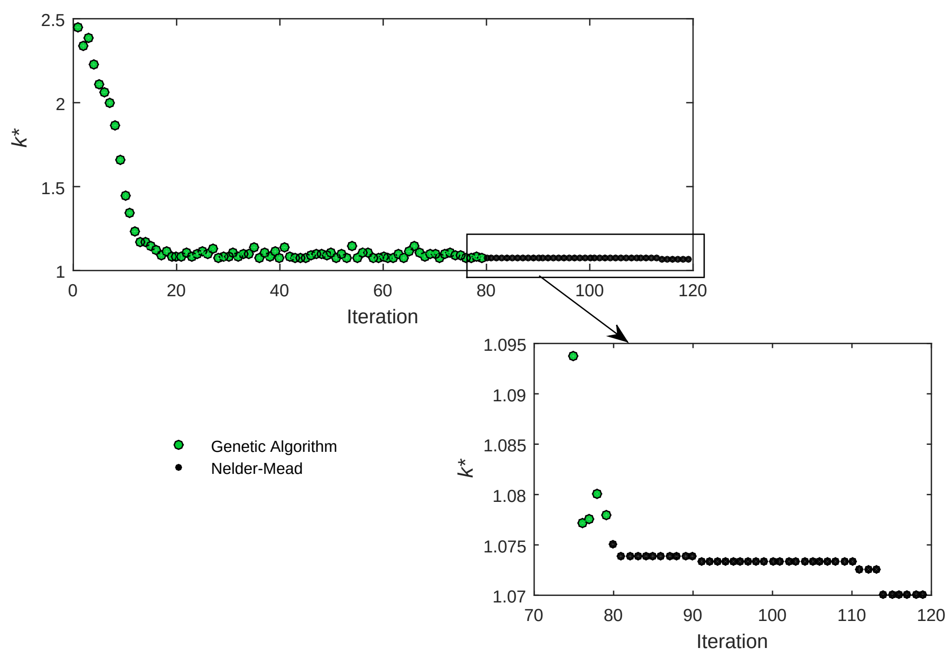

FIGURE 4.4: Convergence to the optimal solution with Genetic Algorithm and Nelder-Mead direct search

In Figure 4.4, the green dots represent the actual values of the objective function per iteration. In addition, the black dots denote the convergence of TC in the second phase when Nelder-Mead direct search was applied. The hybrid optimization allowed to achieve an additional 0.5% cost reduction.

Table 4.5 summarizes the simulation results.

TABLE 4.5: Results of the simulation (RB VSSI $\overline{X}$ chart)

	n_1	n_2	h_1	h_2	k^*	w^*	$TC(10^6)$	$\Delta C(\%)$
Initial state	2	4	2	1	3.000	2.000	1.236	–
Optimization: GA	2	4	2	1	2.298	2.287	1.075	13.0
Optimization: GA+NM	2	4	2	1	2.298	2.175	1.070	13.5

The total cost of decisions is reduced by 13.5 % when RB VSSI $\overline{X}$ chart was applied. The achievable total decision cost reduction was nearly 5% in the case of RBT^2 chart as presented by Section 4.2. Based on the results we can say that TC can be reduced more effectively in the case of adaptive control chart. In order to explain this outstanding reduction rate, Figure 4.5 is provided where I illustrate an interval from the time series of the real/detected sample means to compare the patterns of the risk-based and traditional VSSI $\overline{X}$ charts.

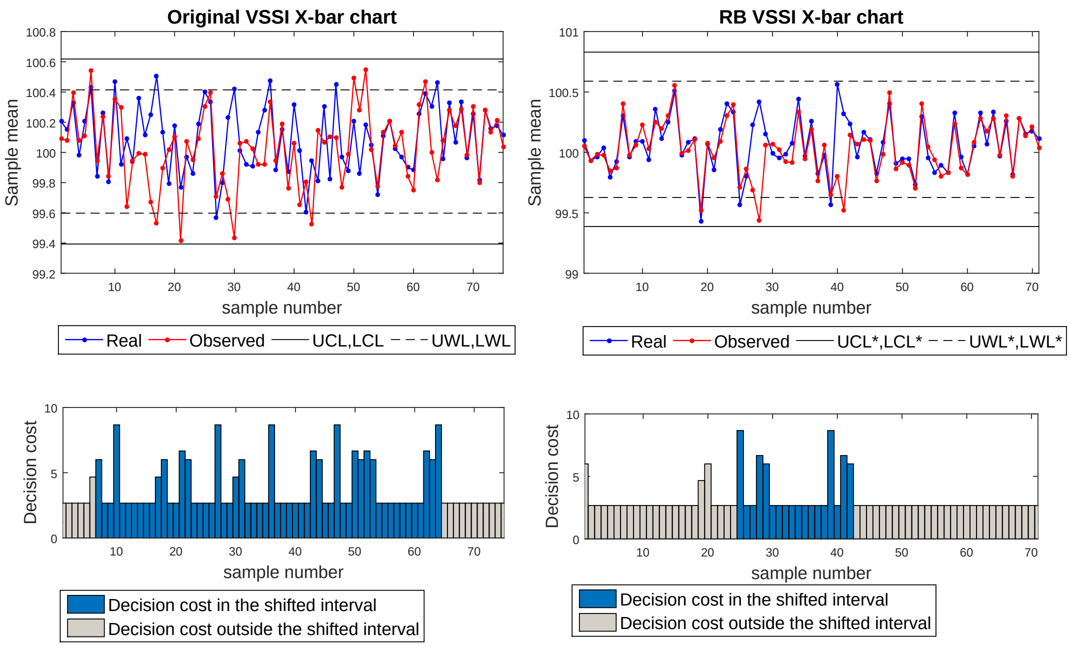

FIGURE 4.5: Comparison of traditional and RB VSSI control chart patterns

Traditional VSSI control chart is shown in the upper-left corner of Figure 4.5, where the control and warning lines were set to their initial values (measurement uncertainty was not considered). The bar chart in the lower-left corner shows the cost value assigned to each decision (to each sampling event). Similarly, the right side of the chart shows the pattern when RB VSSI $\overline{X}$ chart with optimized w and k taking the measurement uncertainty into account.

In the case of the adaptive control chart, the chart pattern depends not only on the values of control limits but also on the width of the warning interval. Sample size and sampling interval are chosen according to the position of the observed sample mean and warning limits. Therefore, the distorting effect of the measurement error can create considerably different scenarios related to the chart patterns resulting differing sampling policies. If the observed sample mean falls within the warning region and the real sample mean is located within the acceptance interval, the sample size will be increased and sampling interval will be reduced incorrectly, leading to increased sampling costs. In the opposite case, sampling event is skipped, which delays the detection of the process mean shift.

As it is demonstrated by Figure 4.5, when the traditional VSSI chart is applied, the two process patterns (observed and real) become separated from each other by the 7^{th} sampling. Incorrect sampling policy is used due to the effect of measurement errors, causing strong separation of the two control chart patterns.

On the other hand, RB VSSI $\overline{X}$ chart takes measurement uncertainty into account and modifies the warning interval, enabling better fitting of the two control chart patterns. The shifted interval is denoted by blue colored columns on the lower bar charts. The charts show that the RB VSSI $\overline{X}$ chart reduces the length of the "separated" interval. In other words, the proposed method not only reduces the total decision costs regarding out-of-control state but also rationalizes the sampling policy. Therefore, greater decision-cost reduction can be achieved in adaptive case compared to the results of Section 4.2.

As a significant contribution, this study also raises awareness of the importance measurement uncertainty in the field of adaptive control charts.

Conclusion of the analysis:

The risk-based aspect can be used to reduce the overall decision cost by adaptive control chart. Compared to RBT^2 chart, RB VSSI $\overline{X}$ chart is more powerful in cost reduction since the proposed method is able to eliminate the incorrect decisions related to the sampling strategy as well.

Chapter 5

Sensitivity Analysis

In this chapter, several simulations are provided to analyze the sensitivity of the proposed methods. The same structure is followed: Section 5.1 includes analyses regarding optimized conformity control procedure, Section 5.2 and 5.3 investigates the sensitivity of the proposed risk-based multivariate and adaptive control charts.

5.1 Characterization of measurement error distribution

In this section, two simulations are provided that demonstrate:

1. sensitivity of K^* towards the change of each decision cost (c_{11}, c_{01}, c_{10}, c_{00})
2. sensitivity of K^* regarding process performance (P_{pk})

5.1.1 Sensitivity analysis for decision costs

In this subsection, K^* was evaluated for different levels of each decision cost in order to analyze their relationship. Since there are four decision outcomes, the simulation has four scenarios, where the chosen decision cost was modified and other three were fixed (ceteris paribus). Figure 5.1 presents the results of the simulations.

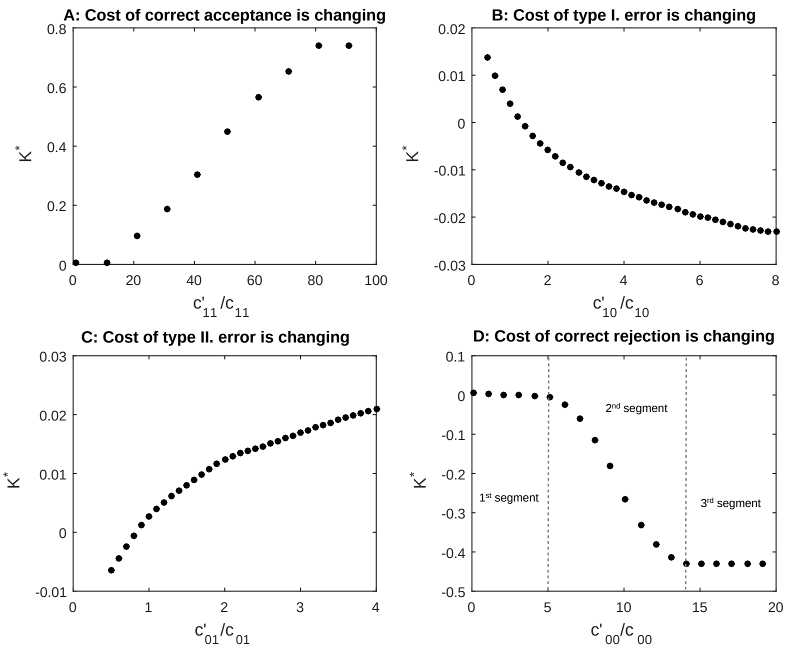

FIGURE 5.1: Sensitivity analysis for cost of each decision outcome

Horizontal axis shows the relative costs where c'_{ij} is the modified cost and c_{ij} is the initial cost during the simulation, e.g., c'_{11}/c_{11} denotes the ratio of the modified decision cost with regard to its original value. During the simulation, only LSL was considered as it was presented by Subsection 4.1.

Case "A" on Figure 5.1 presents the behavior of K^* when c_{11} is changed ceteris paribus. If c_{11} increases, K^* also increases (i.e. the acceptance interval is narrowing) because of the additional cost of the acceptance/production.

In Case "B", the tolerance interval expands (K^* decreases) while cost of type I. error increases because rejection of a conforming product considerably enlarges TC. The pattern has decreasing slope (the optimum value barely changes after eight-fold cost increase) because with the expansion of the acceptance limit, the amount of the measured values outside the region is approaching zero.

Case "C" shows K^* as a function of the cost of type II. error. The curve shows increasing trend: more stringent acceptance rule is applied to reduce the probability of incorrect acceptance.

Case "D": The behavior of K^* can be divided into three segments. In the first segment, K^* slightly reacts to the change of c_{00} because the effect of type II. error is stronger and the method does not increases the acceptance interval. In the second segment, the method increases the acceptance interval (while decreases K^*) in order to avoid correct rejections regardless of the cost of other decision errors. If c_{00} is extremely high (third segment), the method increases the acceptance interval as much as possible but after a certain point there is no sense of further modification since there are no data points below LSL. Let us note that this situation is very unreal in practice.

5.1.2 Sensitivity analysis for process performance (P_{pk})

If the deviation of the controlled process is small enough, the measured value of the product characteristic never reaches the specification limit. The aim of this simulation is to answer the question where is the point regarding process performance, where the consideration of measurement uncertainty can still decrease the total decision cost. In other words, where is the limit, beyond that measurement uncertainty has no impact to the total decision cost. This analysis also determines the limitations of the proposed method.

In the simulation, the real value of the measurand is centered to the target value, and process performance (P_{pk}) is modified with the alteration of the standard deviation. Figure 5.2 presents the result of the simulation. Six scenarios were simulated with different cost structures:

- Case A: General cost structure
- Case B: Type I. error cost is twice as type II. error cost
- Case C: Cost of type II. error is increased significantly
- Case D: Cost of correct rejection is enlarged
- Case E: Cost of correct acceptance is high
- Case F: Cost of type II. error is extremely high

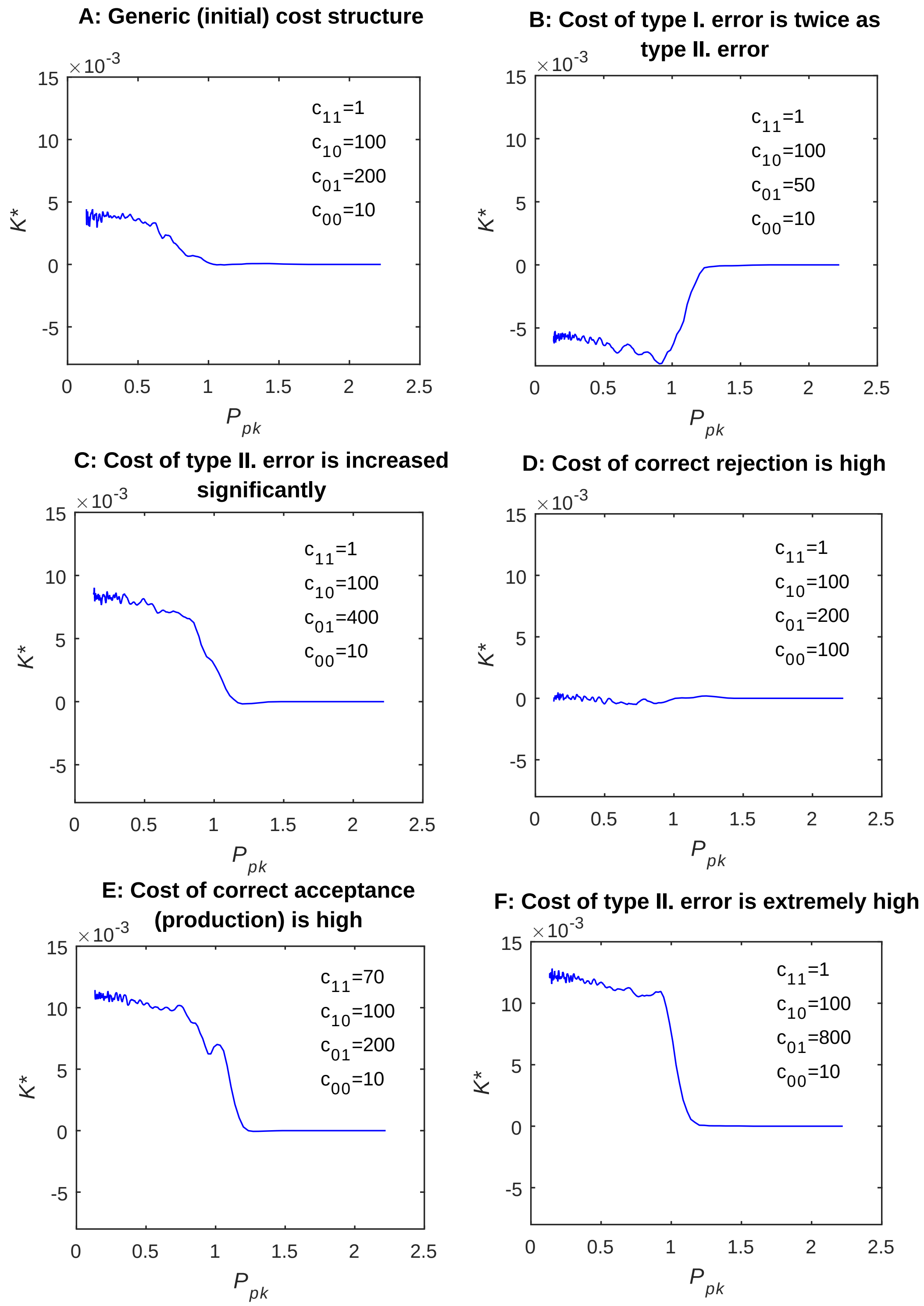

FIGURE 5.2: Sensitivity analysis for process performance (P_{pk})

For cases A, C, E and F, the same phenomena can be observed. When the P_{pk} is low, K^* fluctuates around a given value based on the cost structure and measurement uncertainty. $K^* > 0$ because the most significant cost is the cost of type II. error; therefore, the acceptance interval must be narrowed. If P_{pk} improves, the value of K^* decreases dynamically. Finally, when P_{pk} approaches approximately 1.3-1.5, the correction component tends to zero, meaning that this is the limit where the measurement uncertainty does not have any impact on the decisions.

Case B is quite similar but shows reverse pattern. Initial value of K^* is negative because the dominant cost is the cost of type I. error (acceptance interval needs to be extended). K^* tends to zero when P_{pk} approaches 1.3-1.5 as in cases A, C, E and F. The model is sensitive to the process performance when P_{pk} is low however, it can not find better solution for K in the case of a process with strong performance, because no significant amount of scrap arises.

This analysis showed that the benefit of measurement uncertainty consideration strongly depends on process performance. Nevertheless, it can be assumed that the standard deviation of measurement error can influence this statement. Therefore, this sensitivity analysis was extended with an additional variable. As a further step, the value of K^* was investigated as the functions of P_{pk} and standard deviation of measurement error distribution (σ_ε) compared to the standard deviation of the process (σ_x) (Figure 5.3).

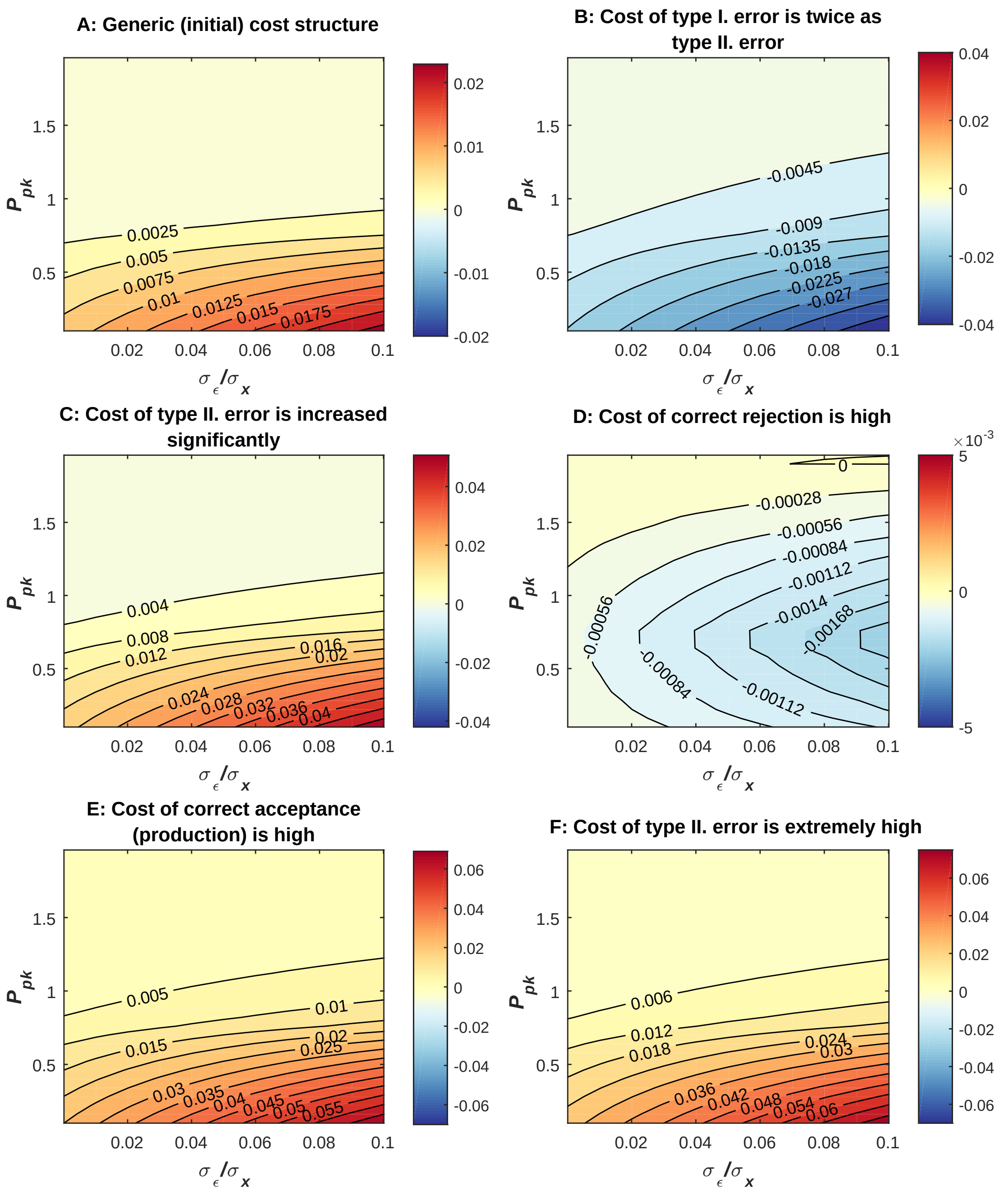

FIGURE 5.3: Sensitivity analysis for process performance (P_{pk}) and standard deviation of measurement error

On Figure 5.3, axis "x" shows the ratio of measurement error- and process standard deviation ($\sigma_\varepsilon/\sigma_x$), axis "y" denotes the process performance and each contour map area represents the value of K^* at each combination of $\sigma_\varepsilon/\sigma_x$ ratio and P_{pk} value. The same cost structures were applied to each simulation (A-F) as it was introduced by Figure 5.2. It is clearly observable that not only process performance but also the ratio of measurement error- and process standard deviation impacts the optimal acceptance strategy. The relationship between P_{pk} and K^* remains the same in all the cases however, this analysis highlights that standard deviation of measurement error (compared to process standard deviation) has significant impact on K^*. That is to say, process performance is not enough to judge the limitation of the proposed method because it can find better solution even at strong process performance if $\sigma_\varepsilon/\sigma_x$ ratio is high. In order to decide if measurement uncertainty is beneficial to consider, P_{pk} and $\sigma_\varepsilon/\sigma_x$ ratio must be taken into account together.

Conclusion of the analysis:

In the case of processes with strong performance index, the consideration of measurement uncertainty cannot decrease the overall decision cost since practically no measured value can be observed near to the acceptance limit. Nevertheless, the benefit of measurement uncertainty consideration can be judged through the joint investigation of P_{pk} and $\sigma_\varepsilon/\sigma_x$.

5.2 Risk-based multivariate control chart

During the sensitivity analysis, the effect of the following parameters are analyzed:

- Decision cost of type II. error (c_{01})
- Sample size (n)
- Skewness of the probability density function of product characteristic 1 (γ_1)
- Standard deviation of product characteristic 1 (σ_1)
- Standard deviation of the measurement error (σ_ε)
- Number of controlled product characteristics (p)

These parameters were chosen, because they can strongly influence the applicability of the proposed control chart. The cost of type II. error may determine the stringency of the control policy. Furthermore, the applicability of the RBT^2 chart can be analyzed under non-normality by modifying the skewness of the distribution regarding the monitored product characteristic. The performance of the control chart is also analyzed under different sample sizes and different level of standard deviation of measurement error.

5.2.1 Cost of type II. error

During the simulation, TC and K^* are evaluated under different levels of c_{01} (while c_{01} is changed ceteris paribus). The same process was considered and same input parameters were used as provided by Subsection 4.2 (except for c_{01}, since its value was modified in this simulation). The results are presented by Figure 5.4.

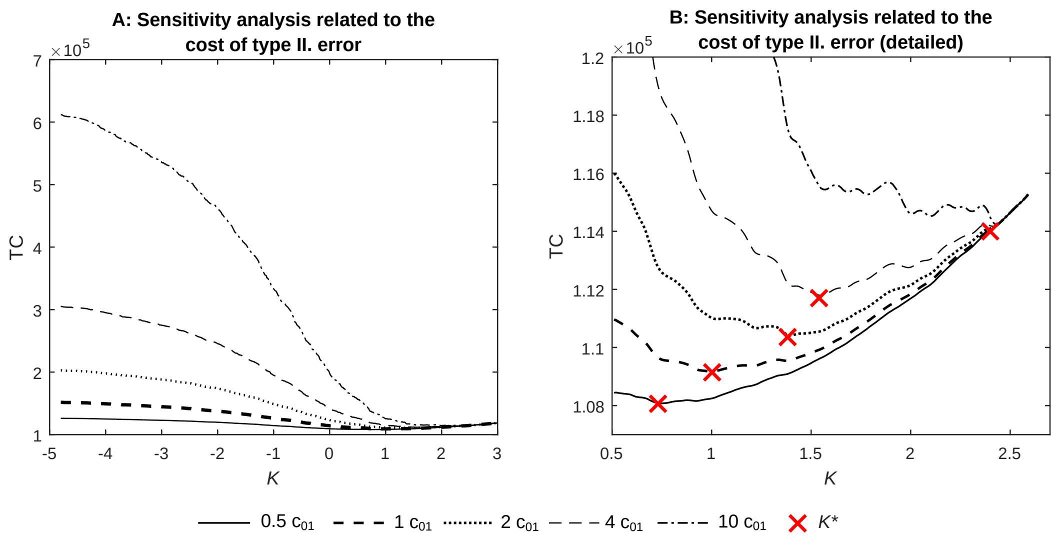

FIGURE 5.4: Sensitivity analysis according to the cost of type II. error

Figure 5.4A shows TC as a function of K for the different levels of c_{01} represented by multiple lines. On the right chart (chart B) more detailed view is provided to highlight location of K^* values. If the cost of type II. error increases, the achievable cost reduction rate is higher and K^* increases (control limit (UCL_{RBT^2}) decreases meaning more stringent control policy).

The results showed that cost of type II. error can strongly influence the optimal value of the acceptance interval however, the proposed RBT^2 chart performs better under enlarged cost regarding type II. error.

5.2.2 Sample size

The sample size is an important question during the application of the control charts. Greater sample size gives better estimation about the production process. On the other hand, greater sample size increases sampling costs especially in case of destructive measurement.

In this simulation, the applicability of the RBT^2 chart is analyzed when different sample sizes are chosen. Figure 5.5 shows the relationship between TC and K under different sample sizes.

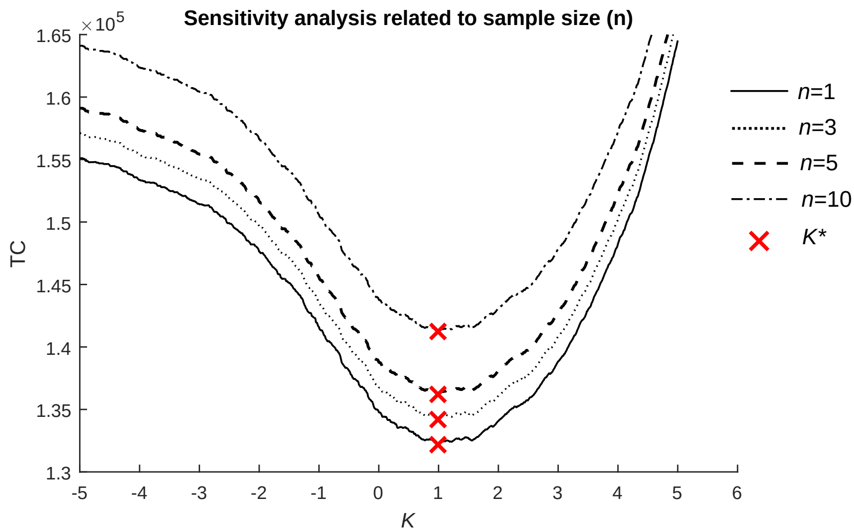

FIGURE 5.5: Sensitivity analysis according to the sample size

With the optimization of the control line, 4-6% total cost reduction can be achieved regardless of sample size. Furthermore, K^* does not react significantly to the change of sample size meaning that RBT^2 chart is not sensitive to the sample size.

5.2.3 Skewness of the probability density function

Subsection 4.2 showed that the proposed method can reduce the total decision cost if the product characteristics follow normal distribution. All the input parameters are the same as they were defined in the simulation results in Subsection 4.2. Although in this case the 3^{rd} moment (skewness) of the distribution function related to the product characteristic 1 is modified and all the other moments remain constant. In this sensitivity analysis I investigate the relationship between K^* and γ_1 (skewness of distribution function related to the product characteristic 1). Figure 5.6 shows K^* as function of γ_1.

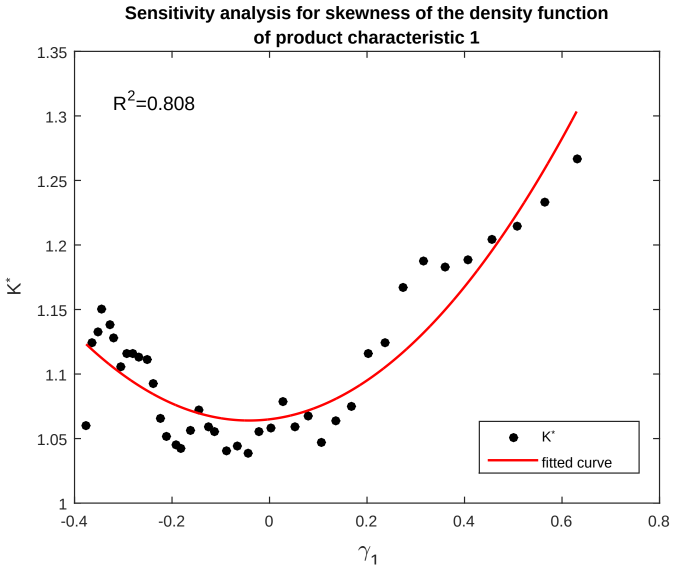

FIGURE 5.6: Sensitivity analysis according to skewness of product characteristic 1

The results show that K^* is sensitive to γ_1. For left- or right skewed distributions, the model returns higher correction component. The pattern also highlights that K^* can be found even under non-normality that provides lower total decision cost. Based on the results, I can conclude that the proposed RBT^2 chart can be used under non-normality (because the control line is calculated by optimization and not analytically with the assumption of a given distribution type), however, skewness affects the optimal correction component.

5.2.4 Standard deviation of process and measurement error

This simulation was conducted considering two different scenarios:

1. Standard deviation of product characteristic 1 was modified ceteris paribus (in every iteration) in order to investigate the relationship between K^* and σ_1
2. Standard deviation of measurement errors (σ_ε) was modified ceteris paribus (in every iteration) to analyze the behavior of K^*

In case of process standard deviation, σ_1' denotes the actual and σ_1 represents the initial standard deviation. In the second phase of the simulation, measurement error standard deviation is expressed as the ratio of σ_1 ($\sigma_\varepsilon/\sigma_1$). Figure 5.7 shows the results related to the process (Figure 5.7A) and measurement error (Figure 5.7B) as well.

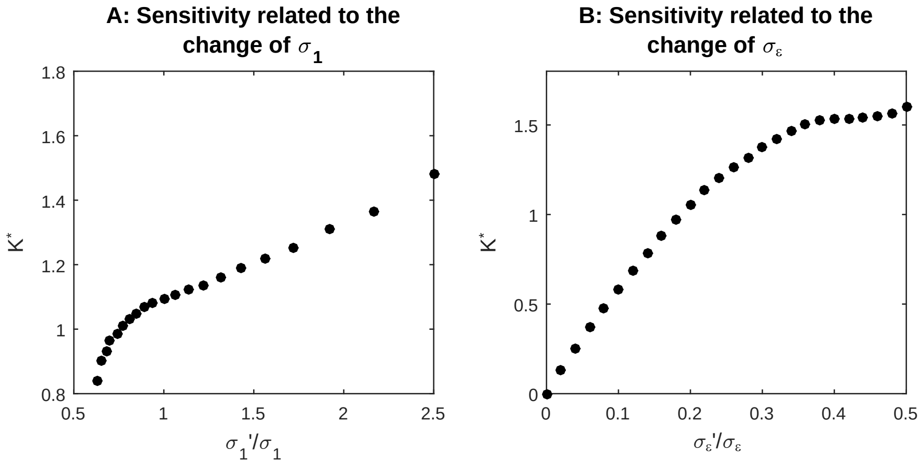

FIGURE 5.7: Sensitivity analysis according to the standard deviation of process and measurement uncertainty

K^* increases in both cases either process or measurement error standard deviation increases. Chart A also confirms the findings of Subsection 5.1.2. It is clearly visible that K^* dynamically drops as σ_1 decreases. With the decrease of σ_1 the process performance improves and the effectiveness of the proposed method deteriorates.

On the other hand, increase of the measurement error standard deviation leads to more stringent control policy as reflected by chart B.

5.2.5 Number of the controlled product characteristics

In the former simulations two controlled product characteristics were assumed, however, it would be beneficial to investigate how K^* changes under different number of controlled product characteristics (p). In order to avoid the effect of the difference between standard deviations and expected values of the simulated product characteristics, the same expected value ($\mu = 25.6$) and standard deviation ($\sigma = 0.07$) were adjusted for each characteristic. In every iteration, the model was extended with an additional product characteristic and K^* was recalculated. All of the characteristics and the measurement error (the measurement error is regarded as constant during the simulation with expected value $\mu_\varepsilon = 0$ and standard deviation $\sigma_\varepsilon = 0.012$) follow normal distribution.

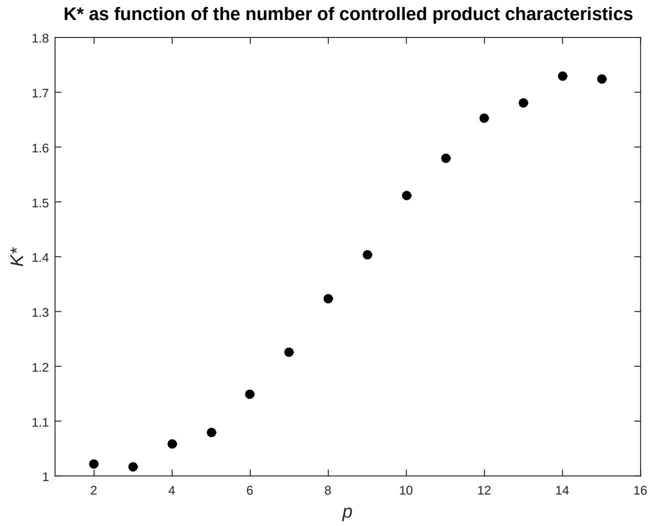

FIGURE 5.8: Sensitivity analysis according to the number of controlled product characteristics

As the results show K^* increases with the number of controlled product characteristics. Though the measurement error is constant, the extension of the product characteristics quantity induces strict control policy, because we increase the number of possible sources regarding uncertainty. Therefore stricter control is applied to avoid the type II. errors since this error type has the highest cost.

5.3 Risk-based adaptive control chart

In this section, I analyze how changes in the cost components, standard deviation and skewness of the measurement error impact the value of k^* and w^*. These factors are selected to assess the limitations of the proposed method. Nevertheless, kurtosis of measurement error could also be examined, Section 4.1 showed that kurtosis does not play significant role in the calculation of the optimal control/specification lines. First, I introduce the analysis regarding to the cost of type II. error.

5.3.1 Cost of type II. error

In order to demonstrate the effect of the cost related to type II. error, the previously defined 9 decision outcomes must be assigned into different groups. This categorization is necessary because there are several decisions out of the 9 outcomes that represents a missed action. Therefore, examining only one of them would be misleading and would provide only restricted information about the sensitivity.

Based on that, I distinguish three groups of decision outcomes:

- Group 1: Type I. error decision outcomes, where the decision is incorrect due to an unnecessary action. Outcomes: #2, #3, #6.
- Group 2: Type II. error decision outcomes, where the decision is incorrect due to a missed action. Outcomes: #4, #7, #8.
- Group X: The remaining decision outcomes, including the correct decisions. Outcomes: #1, #5, #9

During the sensitivity analysis, each cost in Group 2 is multiplied by a changing coefficient (a). Thus, the i^{th} cost is calculated as:

$$C_{4_i} = a_i \cdot C_{4_{initial}} \tag{5.1}$$

$$C_{7_i} = a_i \cdot C_{7_{initial}} \tag{5.2}$$

$$C_{8_i} = a_i \cdot C_{8_{initial}} \tag{5.3}$$

where $C_{4_{initial}}, C_{7_{initial}}, and C_{8_{initial}}$ are the initial values of the decision costs related to cases #4, #7, and #8. a_i is the value of the coefficient in the i^{th} iteration within the simulation, and $i = 1, 2, 3...n$, $i \in \mathbb{N}$ where n is the total number of runs. Figure 5.9 shows the optimal values of k and w (denoted by k^* and w^*) as a function of the cost multiplicator a.

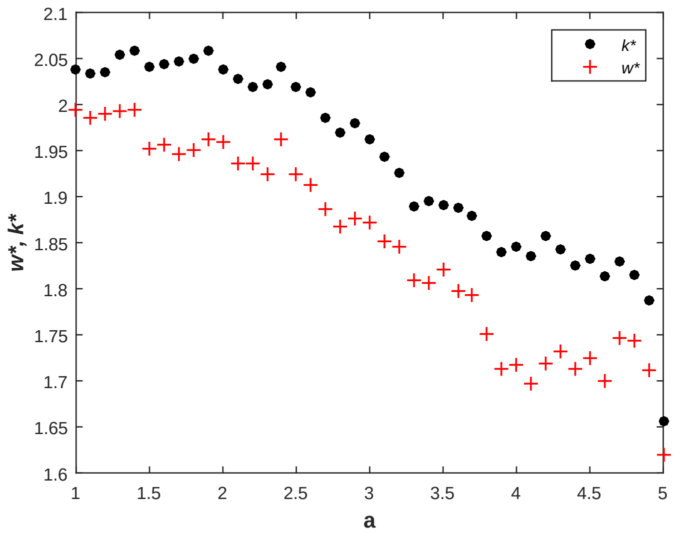

FIGURE 5.9: Sensitivity analysis regarding type II. error - related cost components

On Figure 5.9, black dots represent the values of k^* and red crosses indicate the values of w^*. While C_4, C_7 and C_8 increase, the optimal values of both, k and w decrease. The increase of a, makes the control policy stricter and control and warning limits must be moved closer to the central line to avoid type II. errors. The increase in type II.-related costs does not have considerable impact on the width of warning region (also represented by the distance between k^* and w^*). As a increases, k^* and w^* move to the same direction simultaneously.

To further analyze the behavior of the warning interval, additional sensitivity analysis was conducted based on the sampling cost because it directly influences the warning limit coefficient. Figure 5.10 shows the results of the analysis.

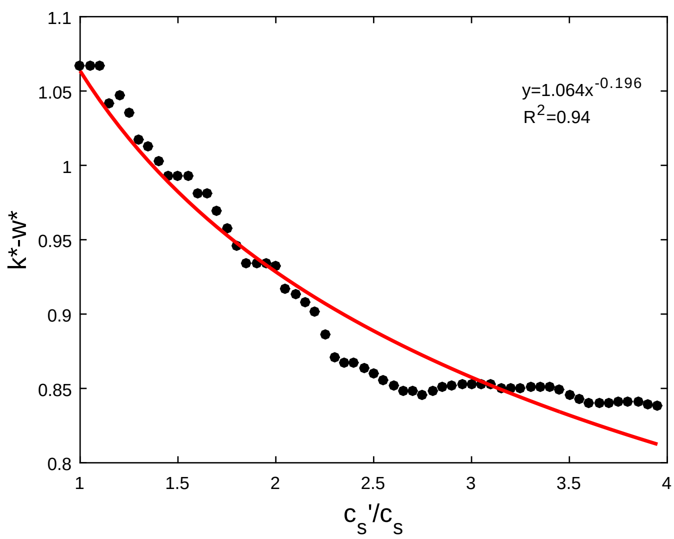

FIGURE 5.10: The width of warning interval as a function of sampling cost

Figure 5.10 shows the distance between k^* and w^* $(k^* - w^*)$ as a function of the sampling cost (c_s). The higher the cost of sampling is, the smaller is the distance between the two limits (width of central region increases). Higher sampling cost increases the value of w^* because the control is too expensive due to the frequently enlarged sample size and shorter sampling interval. On the other hand, lower sampling cost allows stricter control policy.

5.3.2 Standard deviation of measurement error

Sensitivity analysis according to the standard deviation of the measurement error is performed in this subsection. All the distribution parameters were fixed during the simulation except the standard deviation of the measurement error (σ_ε).

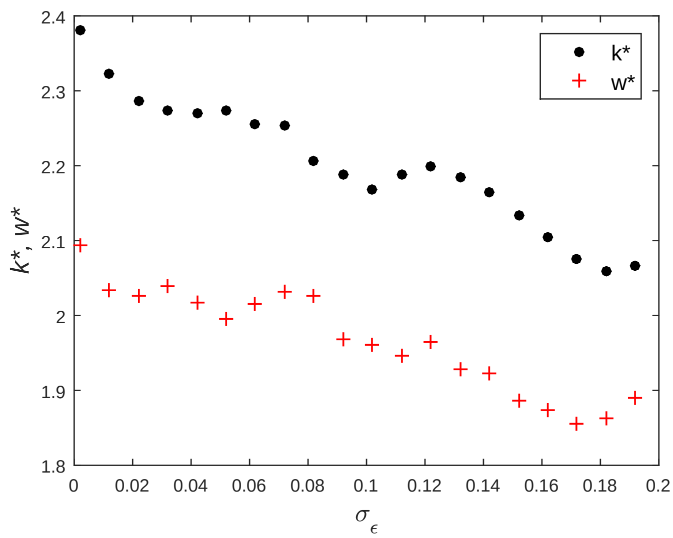

FIGURE 5.11: Sensitivity analysis regarding standard deviation of measurement error

It is observable on Figure 5.11, that w^* and k^* decrease as the standard deviation of the measurement error increases. Higher standard deviation (according to the measurement error) represents a stricter control policy. In this case, the effect of measurement uncertainty is significant; therefore, the approach reduces the width of the control interval to avoid type II. errors. The distance between the two limits is nearly constant because the sampling cost does not change during the simulation. Nevertheless, the sampling cost has considerable impact on the distance between w^* an k^*, as it was shown by Subsection 5.3.1.

5.3.3 Skewness of the measurement error

Since it was proved in Section 4.1, the kurtosis of the measurement error distribution does not impact the control line value, the current sensitivity analysis focuses on the skewness of the measurement error distribution only.

In the simulation, the model parameters were the same as in Section 4.3, but the skewness of the measurement error distribution (denoted by γ) was altered in each iteration (from -1 to 1).

Figure 5.12 shows the results of the sensitivity analysis.

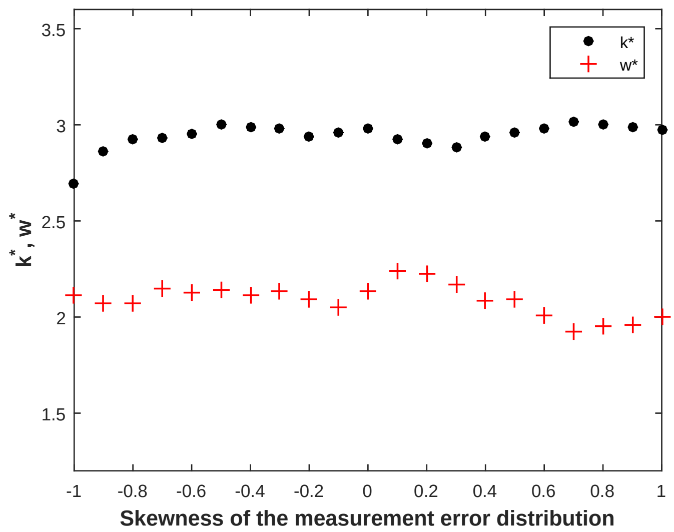

FIGURE 5.12: Sensitivity analysis regarding skewness of measurement error

In the simulation, k^* and w^* are not affected by changes in γ because sampling adjusts the skewed distribution to normal. Based on the central limit theorem, the sample means tend to normal (and the skewness approaches 0).

Chapter 6

Validation and verification through practical examples

In this chapter, I validate the proposed methods and statements through real practical examples with the contribution of a company from automotive industry. The aim of this chapter can be summarized as follows:

1. Validation of the assumption that skewed or asymmetric measurement error distribution is an existing phenomena in production environment.
2. Verify that the proposed methods can simulate and give good approximation to the real process patterns allowing the user to achieve better control policy with the consideration of measurement errors.

In this chapter, three practical examples are presented (the first one is an acceptance sampling example, the second one relates to the T^2 control chart application and third one focuses on the construction of the Risk based VSSI $\overline{X}$ chart) where the "real" (x) and "detected" (y) values are determined by the measurement laboratory of the company. Each example follows the structure below:

1. Selection of the appropriate products and product characteristic(s) that will be analyzed during the experiment.
2. Determination of x and y values using 3D optical scanner from the measurement laboratory
3. Characterization of monitored process and measurement error distribution.
4. Simulation of the selected process and measurement error with the proposed methods using the process/measurement error parameters derived from "step 3".
5. Optimize both, the "real" and "simulated" processes according to the proposed methods (risk-based conformity control, risk-based T^2 chart and risk-based VSSI $\overline{X}$ chart).
6. Comparison of results given by "simulation" and "real" process optimization

For better clarity, I provide detailed description about each step.

1. Selection of products and product characteristics The selected parts were master brake cylinders produced by the manufacturing company. In the examples, two product characteristics were analyzed, the cutting length and the core diameter of the products. Engineers paid very close attention to ensure that the parts were produced by the same machine with the same setup and they were derived from the same batch.

2. Estimation of x and y After the parts have been selected from production, they were transferred to the measurement laboratory of the company. The company uses a 3D optical laser scanner for very precise measurements/experiments, however it cannot be used for process monitoring due to the time-and cost intensiveness of the measurement process. Therefore, manual devices are applied in the production like manual height gauge, and calipers for diameter measurements. In the examples, the 3D optical scanner was used in order to estimate "real" values (denoted by x) of the selected product characteristic, and the devices in the production (manual height gauge, caliper) were used to determine the "detected" value (denoted by y).

Although, the 3D optical scanner has its own measurement uncertainty, it can estimate the "real" product characteristic well, because it is considerably more precise than devices used in the production. The optical scanner was even validated using standard calibration artifact and the average measurement error was lower than 0.001 [mm].

3. Characterization of measurement error After the estimation of x and y values, the measurement error can be estimated as well:

$$\varepsilon_i = y_i - x_i \tag{6.1}$$

Where ε_i is the measurement error related to the i^{th} product. If the measurement error is known for each measurement, its distribution can be analyzed and characterized (mean, standard deviation, skewness, kurtosis).

The same characterization can be done not only for the measurement error, but regarding the process parameters as well (process mean, standard deviation, kurtosis, skewness, trend of the process).

4. Simulation of the process and measurement error Using the information from the previous step, x and ε can be simulated and y can be calculated according to Equation (3.1). The aim of this step is to demonstrate how accurately the monitored process patterns and measurement errors can be simulated.

5.-6. Optimization and comparison of results The aim of this step is to analyze, how efficiently the proposed methods can be used when not all the information are available and simulation needs to be used due to the limitations of the production/measurement system or cost-intensiveness of the measurements.

As a first step, the known process needs to be optimized (x, y and ε are known based on the laboratory measurements). As further step, simulated process is also optimized, and finally, the simulation results (optimal correction component, optimal control limit coefficient) are substituted back to the "real" system allowing us to compare the results given by the real system optimization and optimization results using simulated processes.

In the next section I introduce the first practical example, which is a total inspection problem where the conformity testing procedure is affected by measurement errors.

6.1 Effect of measurement error skewness on optimal acceptance policy

This example introduces a conformity testing problem under the presence of measurement error.

6.1.1 Brief description of the process

The inspected product is a master brake cylinder and the monitored product characteristic is the cutting length [mm] of the product. The acceptance has lower and upper specification limit and the tolerance interval regarding the aforementioned characteristic is 69.25 ± 0.65 [mm] (LSL=68.6 [mm], USL=69.9 [mm]). 50 parts were selected for total inspection and their conformity has to be judged based on the detected cutting length, which is measured by a manual height gauge.

The finance estimated the relative cost of each decision outcome. Cost of correct acceptance (c_{11}) equals 1 and the other decision costs were estimated compared to that. According to that, the outcomes were estimated as follows: c_{11}=1 (correct acceptance), c_{10}=4 (incorrect rejection), c_{01}=34.7 (incorrect acceptance), c_{00}=4 (correct rejection).

6.1.2 Measurement error characteristics

Measurement errors (ε_i) for each measurement were calculated as the difference of the manual height gauge (y_i) and 3D optical scanner measurements (x_i). Figure 6.1 shows the distribution and the Q-Q plot related to the measurement errors.

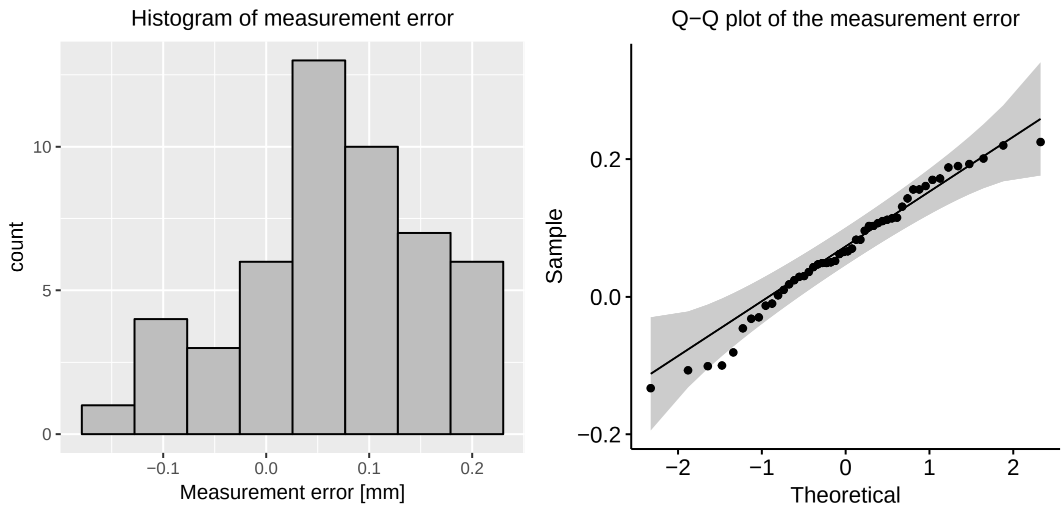

FIGURE 6.1: Distribution of the measurement error (First practical example)

Based on the Q-Q plot, the measurement error follows nearly normal distribution, however the histogram indicates that the distribution is not symmetric. Table 6.1 contains the estimated parameters of the distribution.

TABLE 6.1: Estimated parameters of measurement error distribution (First practical example)

	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Measurement error	−0.13	0.23	0.068	0.089	−0.31	−0.40

The measurement error distribution has negative skewness (−0.31), furthermore, the mean is higher than zero indicating that the height gauge often measures higher value than the real cutting length value. Thus, I expect that during the optimization the proposed method will mainly modify the upper specification limit (USL) in order to eliminate the type I. errors.

As an important contribution, this example confirms that the phenomena of asymmetric measurement error distribution is a valid problem that can be observed in production environment.

6.1.3 Real process and Simulation

In order to simulate the process, first, the parameters of the "real" cutting length- and measurement error distribution must be known. The characteristics of measurement error distribution were already estimated in Subsection 6.1.2, and the parameters regarding the distribution of x were estimated as well (based on the laboratory measurements using the 3D optical scanner):

TABLE 6.2: Estimated parameters of the process distribution (First practical example)

Device	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
3D Optical (x)	68.69	69.89	69.34	0.26	−0.15	3.04
Height gauge (y)	68.71	70.04	69.41	0.32	−0.12	2.57

Both process means are slightly above the target (69.25), which also strengthens the expectation that USL will be affected more by the measurement errors.

In view of the process (x) and measurement error parameters (ε), the simulation can be conducted. For the generation of random numbers with the same distribution parameters, the Matlab's "*pearsrnd*" function was applied.

The result of the simulation compared to the known measurements is introduced by Figure 6.2.

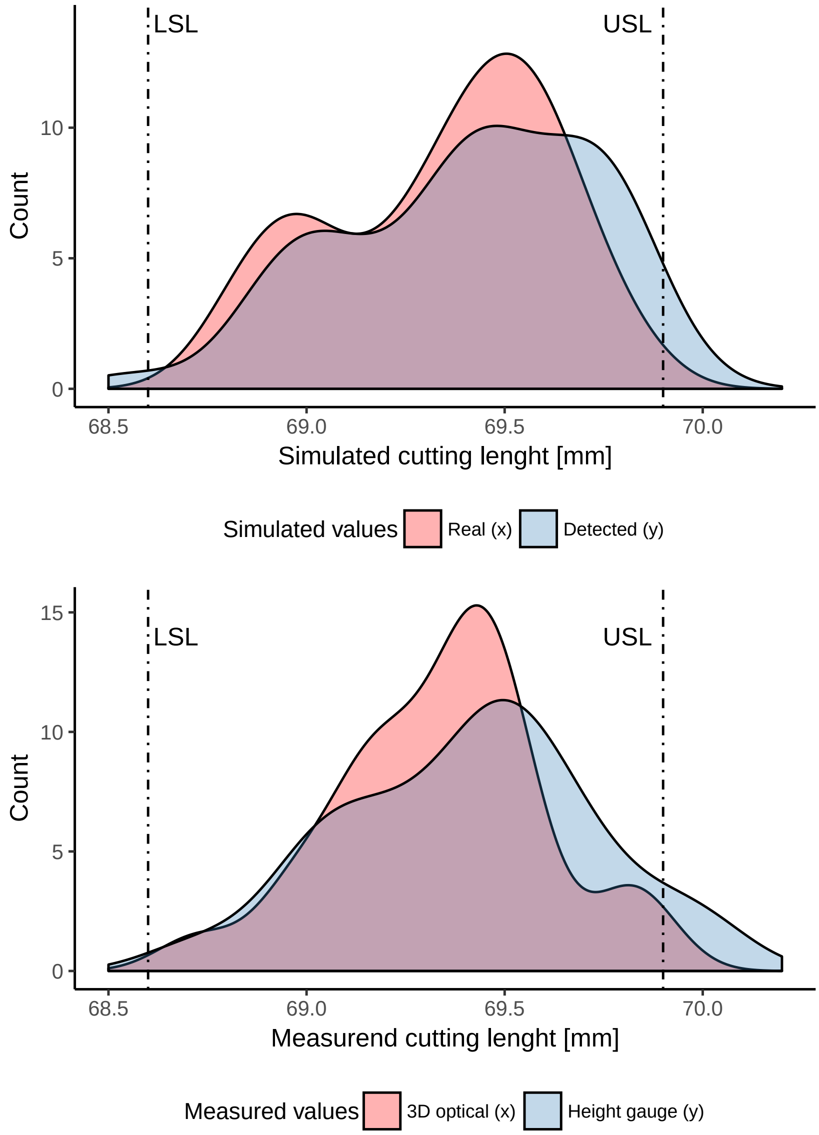

FIGURE 6.2: Density plot according to the real and detected product characteristic values

The upper density plots show the distribution "real" (x) and "detected" (y) product characteristic values when x and y are simulated using the estimated process parameters. The lower chart shows the same density plot, but now x and y are derived from laboratory measurements. (x is derived from the 3D optical scanner measurements and y is the value measured by the height gauge).

Figure 6.2 indicates that the simulation can be used well to describe the characteristics of the real system. In both cases it is clearly visible, that the simulated distributions follow very similar patterns as the measurements derived from the laboratory experiment.

6.1.4 Optimization and comparison of results

Optimization and comparison of results were conducted through the following steps:

1. Total decision cost was calculated using the initial specification limits.

2. Known process (where x is the 3D optical measurement and y is the height gauge measurement) is optimized and total decision cost is calculated.

3. Simulated process was optimized and the resulted correction components were substituted back to the real process, and total decision cost was calculated using these results.

4. Optimization results from simulation and known process are compared in terms of decision cost reduction rate and optimal correction components.

Figure 6.3 shows the density function according to 3D optical measurements (x) and height gauge (y) as well. Black vertical lines represent the initial specification limits, while blue lines represent the optimal specification limits given by the optimization of simulated x and y. Finally, red dashed lines denote the optimized specification using the known data.

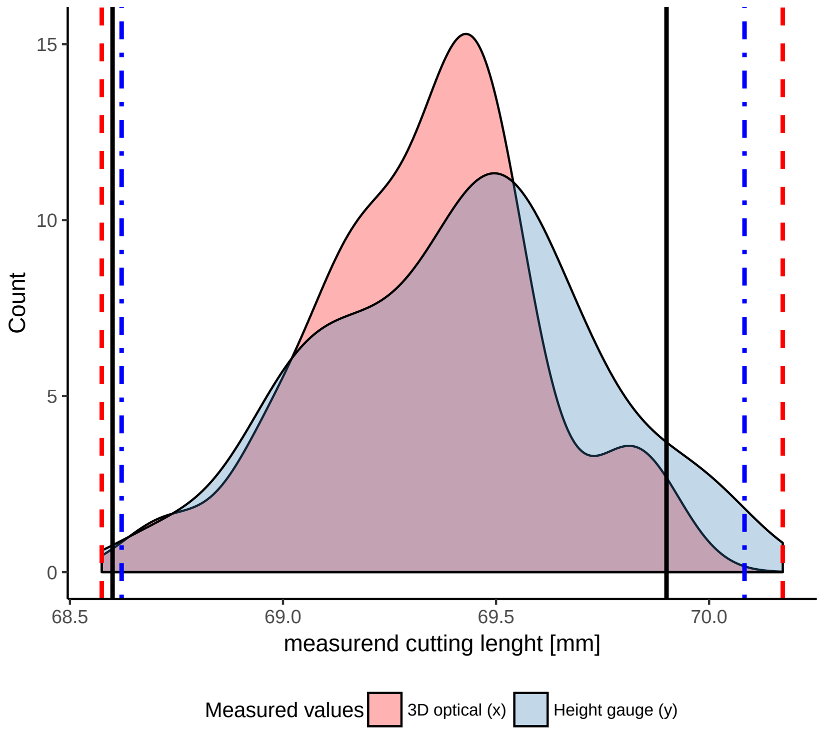

FIGURE 6.3: Density plot with original and optimized specification limits

According to the expectations, there was no significant modification related to LSL due to the negative skewness of the measurement error distribution. Although the alteration of USL is different in the case of simulated and known data, both optimization increased the value of the upper specification limit in order to decrease the number of type I. errors.

Table 6.3 shows the results of the optimizations.

TABLE 6.3: Optimization results (First practical example)

	K_{LSL}	K_{USL}	q_{11}	q_{10}	q_{01}	q_{00}	TC_0	TC_1	$\Delta C\%$
Without optimization	0.00	0.00	46	4	0	0	62	62	–
Optimization using simulated data	−0.02	−0.18	49	1	0	0	62	53	15%
Optimization using known data	0.02	−0.27	50	0	0	0	62	50	19%

K_{LSL} and K_{USL} are the correction components related to LSL and USL respectively. Note that LSL^*, USL^* are the optimal specification limits:

$$LSL^* = LSL + K_{LSL} \text{ and } USL^* = USL - K_{USL} \tag{6.2}$$

Furthermore, q_{11} is the number of correct acceptances, q_{00} is the number of correct rejections, q_{10} denotes the number of type I., while q_{01} represents the number of type II. errors. TC_0 and TC_1 show the total decision costs before and after optimization, finally, $\Delta C\%$ denotes the achieved cost reduction rate.

As the results show, 15% cost reduction rate could be achieved with the elimination of 3 type I. errors if simulated data were used. Additional 4% cost reduction could be achieved if x and y were known (all four type I. errors could be eliminated).

This practical example not only validated that skewed measurement error distribution can exist in production environment but also verified that the proposed method is able to decrease the total decision cost even if x and y are simulated using the preliminary knowledge of their distribution parameters.

6.2 Effect of measurement error on T^2 control chart

In this example, a multivariate T^2 chart is designed to joint monitor two product characteristics. The company and the experiment/measurement methodology is the same as in the previous section, however the monitored product is slightly different.

6.2.1 Brief description of the process

Similarly to Section 6.1, the monitored product is a master brake cylinder, however in this case, simultaneous monitoring of two product characteristics is needed. The first characteristic is the cutting length with tolerance 84.45 $\pm$ 0.75 [mm] and the other one is the core diameter with tolerance 58 $\pm$ 0.5 [mm]. In order to measure the cutting length, manual height gauge is used in the production, and the diameter is measured with calipers. The control policy focuses on the process control and does not take the specification limits into account by this example. Therefore, there are four decision outcomes again, those costs were estimated by the finance: c_{11}=1 (correct acceptance), c_{10}=20 (incorrect control), c_{01}=160 (incorrect acceptance), c_{00}=5 (correct control).

6.2.2 Measurement error characteristics

Measurement errors (ε_i) for each measurement were calculated using Equation (6.1). x_i represents the 3D optical measurement in the case of both characteristics, y_i denotes the measurement given by height gauge (by the cutting length) and it is measured by a caliper in the case of the core diameter.

Figure 6.4 shows the distribution and the Q-Q plot of measurement errors related to cutting length and core diameter.

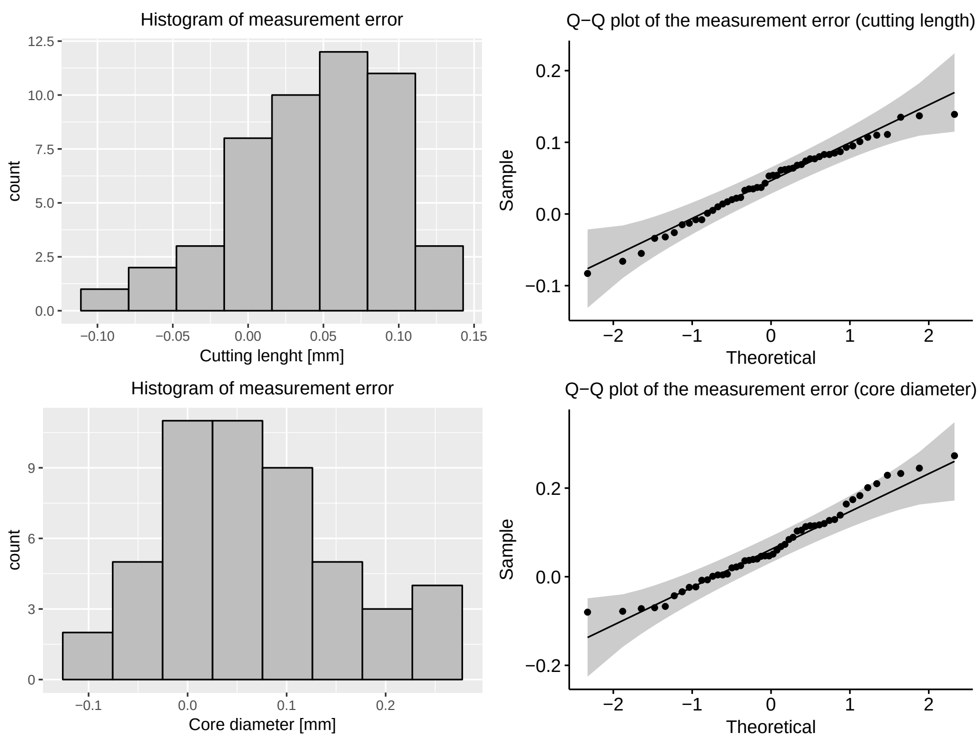

FIGURE 6.4: Distribution of measurement error related to cutting length and core diameter

It is clearly observable on Figure 6.4 that the measurement error distributions have different direction regarding skewness. Table 6.4 contains the estimated parameters of the measurement error distributions.

TABLE 6.4: Estimated parameters of measurement error distribution (Second practical example)

Measurement error	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Cutting length	−0.08	0.14	0.044	0.052	−0.35	2.61
Core diameter	−0.08	0.27	0.067	0.09	0.35	2.42

The estimated parameters also confirm the observed pattern, providing good practical examples for the existence of both, left- and right skewed measurement error distributions in production environment. The skewness is −0.35 regarding cutting length measurement error and 0.35 related to the core diameter measurements. The standard deviation values show that the measurements given by caliper are more distorted. Furthermore, the estimated distribution parameters can be used to simulate measurement errors.

6.2.3 Real process and Simulation

Similarly to Subsection 6.1.3, besides measurement error characterization, process parameters need to be estimated too in order to provide simulated control chart pattern. The process parameters are summarized by Table 6.5.

TABLE 6.5: Estimated parameters of the process distribution (Second practical example)

		Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Length	3D optical	84.30	84.64	84.49	0.07	−0.33	2.96
	Height gauge	84.32	84.74	84.54	0.08	−0.05	3.38
Diameter	3D optical	57.82	58.08	57.89	0.07	1.51	3.99
	Caliper	57.77	58.17	57.95	0.11	0.18	2.12

Although, the processes are well centered, the detected mean values (given be the height gauge and caliper measurements) values are shifted due to the measurement errors. The proposed method can be used under non-normality however, T^2 chart assumes non-correlated product characteristics. Correlation and density functions are shown by Figure 6.5.

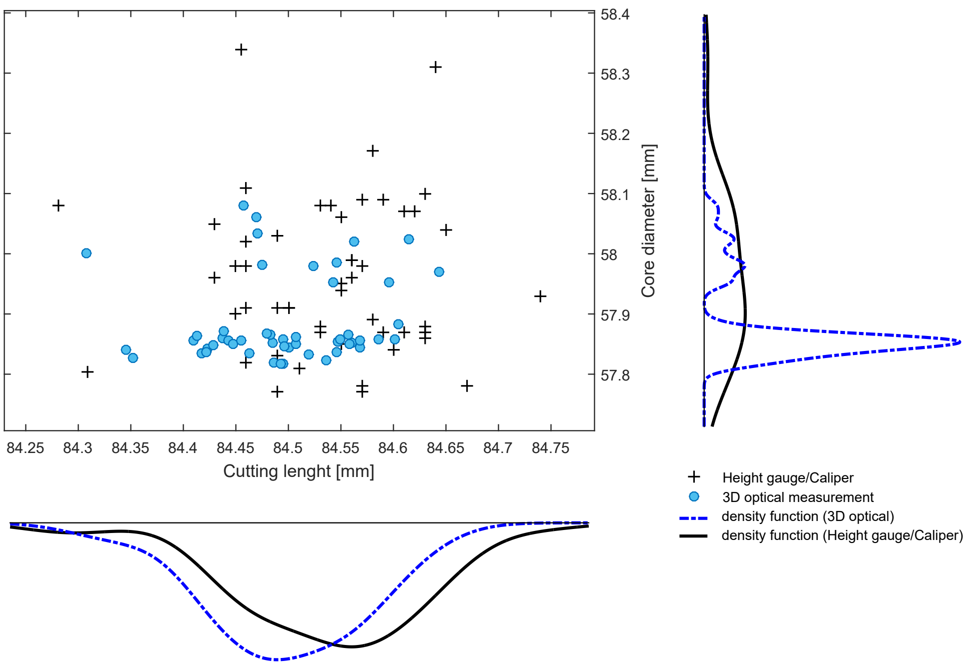

FIGURE 6.5: Correlation and distribution of the two product characteristics

Blue dots are representing the relationship between cutting length and core diameter according to the optical measurements, and black crosses denote the relationship of the product characteristics regarding the manual devices (height gauge or caliper). Based on the scatter plot, no significant correlation can be considered. Pearson correlation coefficients also confirm the observation: $r = 0.24$ $(p = 0.082)$ for the optical measurements and $r = 0.06$ $(p = 0.646)$ for the manual devices.

Density functions also show that the measurement is strongly distorted in the case of caliper.

Since the correlation satisfies the control chart condition, the T^2 chart can be designed and simulation can be conducted. Process mean, standard deviation, information about skewness and kurtosis were used to simulate the processes. Simulated process is introduced by Figure 6.6, where the upper control chart was designed using optical measurement as x and manual device measurements (by height gauge and caliper) as y and the lower chart shows the resulted control chart patterns, when both, x and y are simulated based on the estimated process parameters.

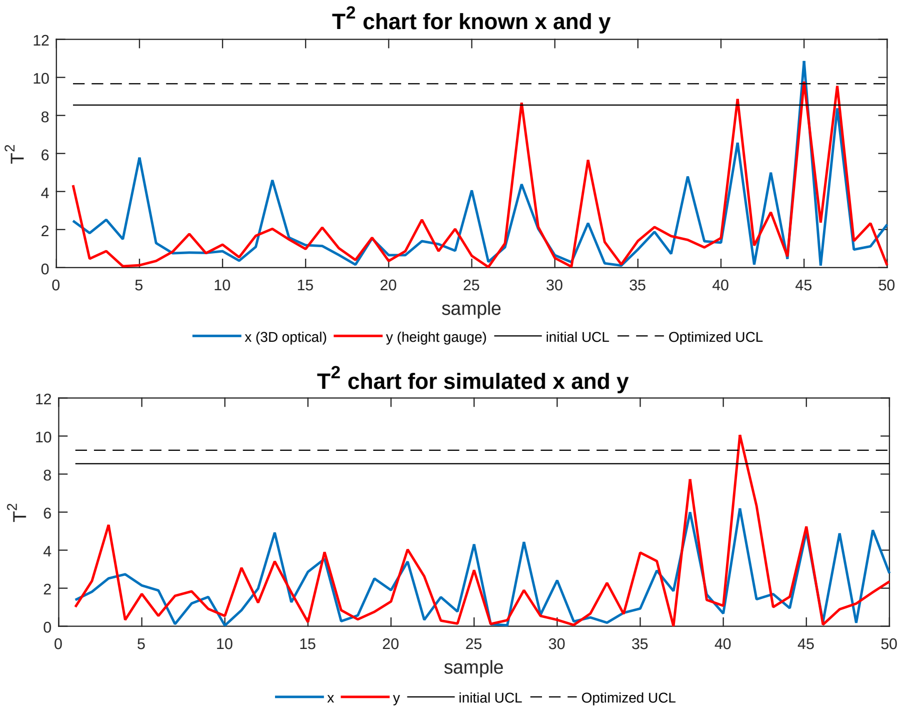

FIGURE 6.6: Designed T^2 charts (upper chart contains the known x and y and lower chart was built under simulated x and y)

Blue lines are representing the "real" values (optical measurements on the upper chart and simulated x values on the lower chart) and red lines denote the "detected" values (manual measurements on upper chart and simulated y on lower chart). As the control chart patterns show, the known process can be modeled well however, the proposed method can be verified only if it can decrease the total decision cost through the optimization of the control limit.

6.2.4 Optimization and comparison of results

Optimization and result-comparison includes the same steps as Subsection 6.1.4, Figure 6.7 shows the optimized control limits.

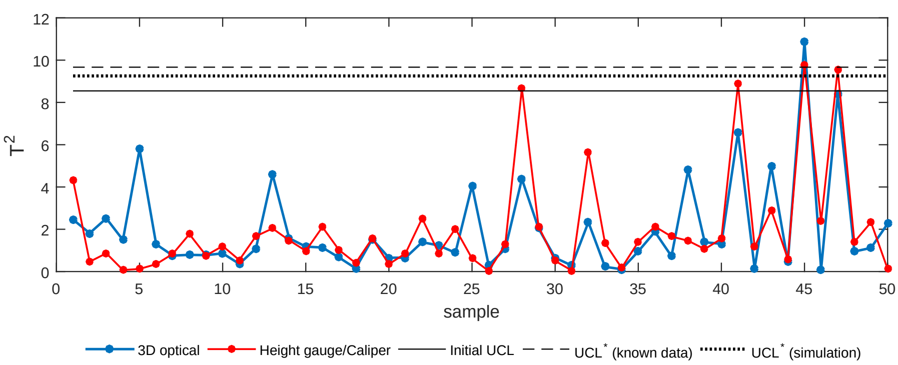

FIGURE 6.7: Designed T^2 chart with optimized control limit

The continuous horizontal line denotes the initial control limit (without optimization), furthermore, dotted and dashed lines represent UCL^* for simulated data and UCL^* for known data respectively (where UCL^* is the optimized value of the control line). Both scenarios increased the acceptance interval in order to eliminate the most type I. errors however, the modification was smaller by the simulated data due to the lack of knowledge. Table 6.6 compares the results of each scenario.

TABLE 6.6: Optimization results (Second practical example)

	K	q_{11}	q_{10}	q_{01}	q_{00}	TC_0	TC_1	$\Delta C\%$
Without optimization	0.00	46	3	0	1	104	104	–
Optimization using simulated data	−0.70	48	1	0	1	104	70	33%
Optimization using known data	−1.12	49	0	0	1	104	53	49%

With the simulated process, total decision cost was reduced by 33% and additional 16% would have been achieved if all the knowledge about x and y would had been available. Please note, that only one correction component (K) can be interpreted here, since T^2 chart has only upper control limit.

The proposed model could find a better solution regarding the control line when the process and measurement error was simulated using the preliminary knowledge about the process and measurement error characteristics. The results verify that the proposed method is able to reduce the decision costs even under restricted information about the real measurements.

6.3 Effect of measurement error on adaptive control chart

The third practical example introduces an adaptive control chart fitting problem and also investigates the applicability of the proposed VSSI $\overline{X}$ chart.

6.3.1 Brief description of the process

The monitored product is a third type master brake cylinder with different serial number. The tolerance related to the product's cutting length is 69.25 ± 0.65 [mm]. 100 parts were selected for the experiment and the measurements regarding x and y were conducted with the same 3D optical device and the manual height gauge. Due to the conditions and limitations of production procedure, sample size cannot be higher than 3 and sample needs to be taken on every hour or in every two hours at most. Therefore, in the control chart design, the variable parameters are considered as $n_1 = 2$, $n_2 = 3$, $h_1 = 2$, $h_2 = 1$, warning- and control limit coefficients (w, k) are optimized.

According to Subsection 3.3.3, nine decision outcomes can be defined and they were estimated by the finance as follows (Table 6.7):

TABLE 6.7: Estimated costs of the decision outcomes

Case	**Estimated relative cost**
# 1	1
# 2	5
# 3	48
# 4	6
# 5	4
# 6	50
# 7	184
# 8	66
# 9	22

For detailed description of each decision outcome see Subsection 3.3.3 and for illustration see Figure 3.2.

6.3.2 Measurement error characteristics

Definition of x, y and ε remained the same as it was defined in the subsections 6.1 and 6.2. Figure 6.8 illustrates the distribution of the data including Q-Q plot as well.

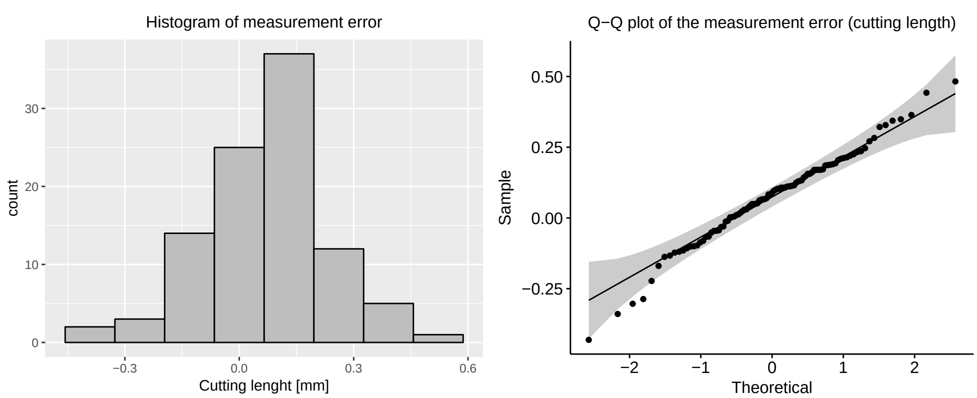

FIGURE 6.8: Distribution of the measurement error (Third practical example)

Based on the histogram and Q-Q plot, the measurement errors follow nearly normal distribution however, the histogram is not symmetric either. The shape of the distribution is similar to the previous examples since the same manual height gauge was applied in the measurements.

TABLE 6.8: Estimated parameters of measurement error distribution (Third practical example)

	Minimum	**Maximum**	**Mean**	**Std. Deviation**	**Skewness**	**Kurtosis**
Measurement error	−0.43	0.48	0.073	0.159	−0.36	3.78

Table 6.8 indicates that the skewness is negative in this case, similarly to the previous cutting length measurements. The estimated parameters are used to generate random errors with the same mean, standard deviation skewness and kurtosis.

6.3.3 Real process and Simulation

In order to simulate x and y, the process distribution parameters need to be estimated too. Table 6.9 contains the estimated moments of the distribution functions related to 3D optical measurements (x) and Height gauge measurements (y).

TABLE 6.9: Estimated parameters of the process distribution (Third practical example)

Device	**Minimum**	**Maximum**	**Mean**	**Std. Deviation**	**Skewness**	**Kurtosis**
3D optical (x)	68.64	69.97	69.36	0.28	−0.14	2.76
Height gauge (y)	68.51	70.24	69.43	0.33	0.01	2.92

The process mean considering the real value of the cutting length is slightly above the target and due to the negative skewness and positive mean of measurement error distribution, y is generally higher than x, which is also reflected by Table 6.9. The process was simulated using the estimated distribution parameters and VSSI $\overline{X}$ chart was designed in order to control cutting length of the product. Figure 6.9 shows the control chart patterns for known and simulated data.

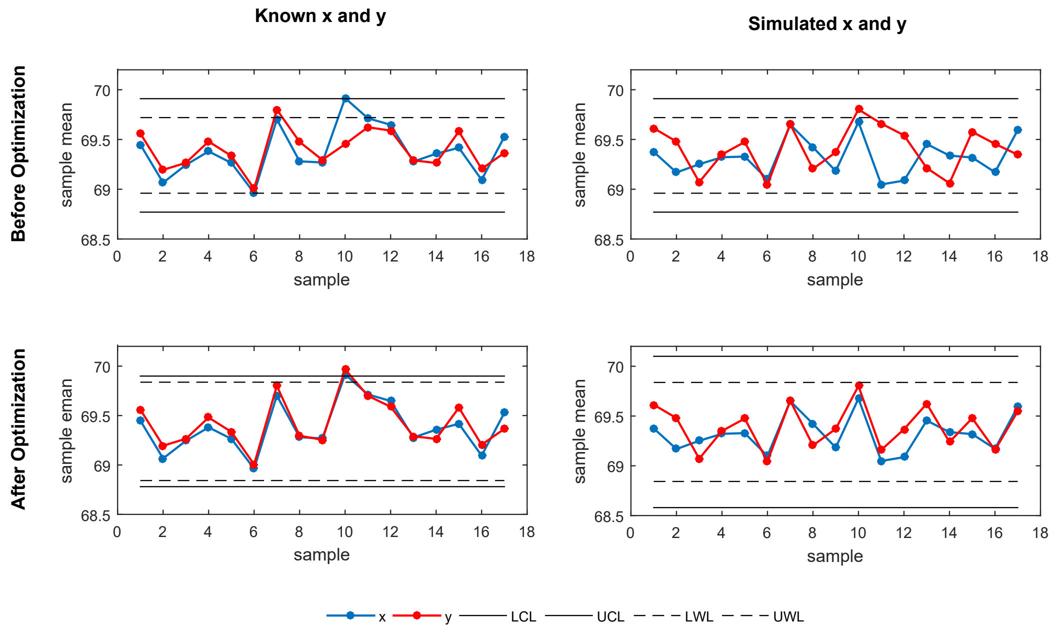

FIGURE 6.9: VSSI $\overline{X}$ chart patterns for known data and simulation

The proposed method is able to optimize the warning limits and thus the real and detected control chart patterns converge better to each other. This leads to cost reduction because sampling policy can be rationalized by eliminating missed sampling events or unnecessary increase of sample size. In both cases (real and simulated processes), the proposed method was able to improve the sampling policy which is clearly visible on the control chart patterns after optimization.

The next subsection summarizes and compares the optimization results related to known and simulated data.

6.3.4 Optimization and comparison of results

During the optimization, w and k were optimized, where w is the warning limit coefficient and k is the control limit coefficient. Table 6.10 shows the quantity of each decision outcome (q_i) and total decision costs as a function of w and k.

TABLE 6.10: Optimization results (Third practical example)

Optimization	w	k	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	TC_0	TC_1	$\Delta C\%$
No optimization	2.00	3.00	15	1	0	0	0	0	1	0	0	204	204	–
Simulated x, y	2.62	3.44	16	0	0	0	0	0	0	1	0	204	82	60%
Known x, y	2.63	2.94	16	0	0	0	0	0	0	0	1	204	38	81%

There were no significant difference between optimized warning limit coefficients however, k^* was higher (3.44 instead of 2.94) than it should have been in order to reach the lowest achievable total decision cost. Both scenarios were able to optimize w and provide better sampling policy through convergence of $\overline{x}$ and $\overline{y}$ control chart patterns. On the other hand, optimized control lines given by simulated x and y were not able to eliminate all incorrect decisions (due to lack of knowledge). As Figure 6.10 shows, at the 10^{th} sample, optimal UCL based on simulation is too high and incorrect acceptance would be made.

Nevertheless, a better control and sampling policy was provided by the proposed method even without the exact knowledge of all data points. 60% cost reduction was achieved when w and k were optimized using simulated measurements and potentially 21% more if all the data points had been known.

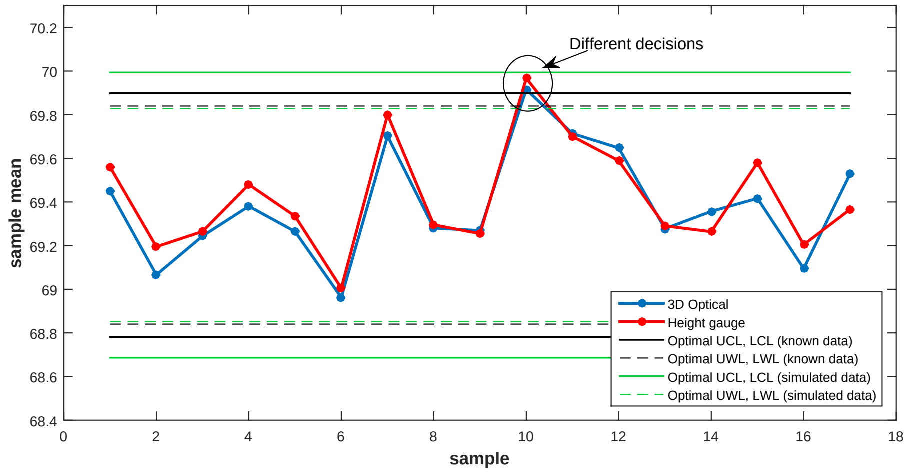

FIGURE 6.10: RB VSSI $\overline{X}$ chart with optimal warning and control lines

It is important to note that much larger cost reduction can be realized because of two reasons:

1. We have only 17 plots on the chart, thus, elimination of a single incorrect decision has huge impact on total decision cost.
2. Rationalization of sampling policy provides further opportunities to decision cost reduction.

This example verified that the proposed method can be a solution to rationalize the sampling and control procedure simultaneously. The RB VSSI $\overline{X}$ chart can be applied in order to reduce total decision cost if process and measurement error parameters and decision costs can be estimated.

Chapter 7

Summary and Conclusion

7.1 Summary

The aim of this work was to broaden the knowledge regarding measurement error distribution and hereby provide a new risk-based control chart design methodology with the consideration of measurement uncertainty. In order to explore the most relevant scientific contributions according to control charts and measurement uncertainty, I conducted a systematic literature review.

As refinement of the literature search, not only relevant papers have been collected but citation database has been built from which citation networks have been constructed. The literature research results confirmed that the concept of measurement error in process control is a significant research area however, decision outcomes should be considered during control chart design and the linkage should be strengthen between measurement uncertainty and control chart design studies.

Chapter 3 introduced the methodology related to the examination of the effect if 3^{rd} and 4^{th} moments of measurement error distribution and the proposed design method for RBT^2 and RB VSSI $\overline{X}$ charts.

Simulation results and several sensitivity analysis were conducted (Chapters 4 and 5) in order to validate research proposals and investigate the performance and limitations of the proposed methods under different conditions. As the outcome of the dissertation, three theses were defined:

Thesis 1: Third moment (skewness) of the measurement error distribution strongly affects the value of the optimal acceptance limit, however fourth moment (kurtosis) of the error distribution does not have significant impact on the acceptance policy when total inspection is applied. In case of acceptance sampling, neither skewness nor kurtosis impacts the optimal acceptance limit due to the central limit theorem. Therefore, in conformity control, measurement uncertainty needs to be considered as distribution with its characteristics and not as an interval. Furthermore, in the case of processes with strong performance index, the consideration of measurement uncertainty cannot decrease the overall decision cost. Relation of process performance and standard deviation of measurement error (compared to process standard deviation) determines if it is beneficial to deal with measurement uncertainty.

Thesis 2: Consideration of measurement uncertainty not only beneficial in the case of Shewhart control chart but can reduce the total decision cost when multivariate control chart is applied.

Thesis 3: The risk-based aspect can be used to reduce the overall decision cost by adaptive control chart. Compared to RBT^2 chart, RB VSSI $\overline{X}$ chart is more powerful in cost reduction since it is not only able to eliminate incorrect decisions with respect to "out-of-control" detection but also reduces the cost related to incorrect sampling policy in "in-control" state.

In Chapter 6, validation and verification of the defined research proposals were introduced. Real practical examples were provided and laboratory experiments were organized to validate the existence of skewed measurement error distribution and verify applicability of the proposed methodology at a company from automotive industry.

7.2 Conclusion

The first contribution of this dissertation is the detailed literature research that not only explores the most relevant studies but models the relationship between control chart design and measurement uncertainty areas.

As an outcome of the literature review, I ascertained that:

1. Many researches aimed to develop methods in order to express the measurement uncertainty even assuming skewed distribution, however there are just a few ones considering the consequences of decisions by conformity control under the presence of non-normal measurement error distribution. On the other hand, the studies dealing with asymmetric measurement error distribution do not investigate the effect of each moments of the measurement error distribution on the effectiveness of conformity control.

2. Although control chart studies proposing multiple sampling strategies in order to reduce the effect of measurement error, they did not consider the risk of the decisions during process control.

3. The linkage between the two research area is weak, only few citations can be observed between the constructed networks.

The literature research results confirmed that the concept of measurement error in process control is a significant research area however, decision outcomes should be considered during control chart design and the linkage should be strengthen between measurement uncertainty and control chart design studies.

As further contribution, this research showed that not only expected value and standard deviation is important during the characterization of measurement error but skewness can strongly influence the performance of the conformity- or process control. It was also reflected by the results that consideration of measurement uncertainty is beneficial in process control. The proposed method not only reduces the number of incorrect decisions but also decreases the total cost associating with the decision outcomes.

The additional implications of this research can be summarized from different point of views: implications for scholars, implications for practitioners and implications for the management.

1. Implications for scholars This dissertation demonstrated how risk-based aspect can be applied for conformity and process control and pointed out that results given by measurement uncertainty researches should be utilized more in control chart design. The dissertation raises attention to the skewness of measurement error distribution indicating that dealing with asymmetric measurement uncertainty is very important in conformity control. The proposed new control charts proved that application of risk-based concept can decrease the decision cost even in multivariate and adaptive statistical process control. My research results were published in the following international scientific papers:

Thesis 1:

Kosztyán, Zsolt T., Csaba Hegedűs, and Attila Katona (2017). Treating measurement uncertainty in industrial conformity control. In: *Central European Journal of Operations Research*, pp. 1-22. ISSN: 1613-9178. DOI: doi.org/10.1007/s10100-017-0469-8

Thesis 2:

Kosztyán, Z. T., & Katona, A. I. (2016). Risk-based multivariate control chart. In: *Expert Systems with Applications*, 62, 250-262. DOI: doi.org/10.1016/j.eswa .2016.06.019

Thesis 3:

Kosztyán, Z. T., & Katona, A. I. (2018). Risk-Based X-bar chart with variable sample size and sampling interval. In: *Computers & Industrial Engineering*, 120, 308-319. DOI: doi.org/10.1016/j.cie.2018.04.052

Figure 7.1 shows the placement of these articles relative to the introduced literature networks. The green edges represent the papers I cited from the network.

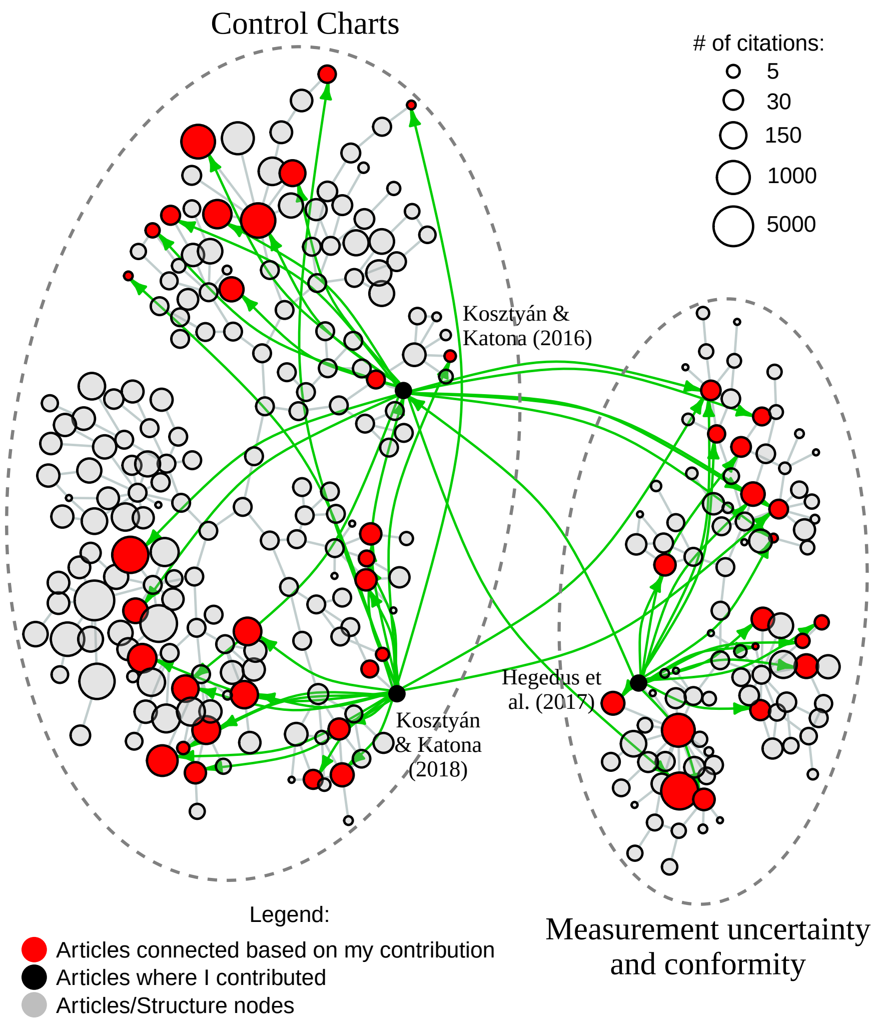

FIGURE 7.1: Placement of the research outcomes into the main stream

2. Implications for practitioners Practitioners can benefit from the outcomes of this work, because the product characteristics can be monitored more efficiently with the proposed risk-based control charts. Process shifts can be detected more precisely in multivariate (RBT^2) or adaptive (VSSI $\overline{X}$) cases as well. In addition, even sampling procedure can be rationalized with the RB VSSI $\overline{X}$ chart. This research also determines the process performance value where it is still beneficial to consider measurement uncertainty.

3. Implications for the management For a manufacturer company, quality of the products is outstandingly important in terms of competitiveness and the proposed risk-based control charts are able to maintain high quality and decrease decision costs in the same time. Quality management can leverage the proposed methods by decreasing the amount of type II. errors (prestige loss), decision costs and increase the overall customer satisfaction.

Appendix A

Risk and uncertainty in production management

Risk and uncertainty are commonly discussed topics on the field of production management. As it have been provided by the Royal Society in 1992, the risk is: "The probability that a particular adverse event occurs during a stated period of time, or results from a particular challenge. As a probability in the sense of statistical theory, risk obeys all the formal laws of combining probabilities". Based on the review of Harland et al., different types of risks can be distinguished (Harland et al., 2003): strategic risk (Simons, 1999), operations risk (Meulbrook, 2000, Simons, 1999), supply risk (Meulbrook, 2000), Smallman, 1996), customer risk (Meulbrook, 2000), asset impairment risk, competitive risk (Simons, 1999), reputation risk (Gibb and Schwartz, 1999), financial risk (Meulbrook, 2000), fiscal risk, regulatory risk (Meulbrook, 2000, Cousins et al., 2004, Smallman, 1996), legal risk (Meulbrook, 2000).

The statistical process control - which is the main topic of the thesis - is directly connected to the operations risk (since the incomplete process control affects the producer's ability to manufacture) and customer risk (because the incorrect control increases the probability of defected product occurrence).

However there are several types of risk, uncertainty is a significant element included by each type of them (Yates and Stone, 1992) and it is associated with the degree of confidence of a decision maker during the decision making procedure. (Mitchell, 1995). Clarkson and Eckert (2010) distinguish four categories of uncertainty: known uncertainties, unknown uncertainties, uncertainties in the data (including measurements) and uncertainties in the description.

Uncertainties that can be handled well based on the knowledge of past cases are the known uncertainties. Unknown uncertainties are the events that could not be foreseen like the occurrence of 9/11 and its impact (Weck and Eckert, 2007). Uncertainties in the data mean the factors like completeness of the data, accuracy, consistence and the quality of the measurement. And finally, uncertainty in the description is the fourth category, which is related to the description of a system and focuses on the ambiguity (or clarity) of the description. There is a significant difference between the last two categories from the point of view.of mathematical modeling. Since the measurement process can be described well with the characteristics of the measurement error the uncertainty of data can be modeled well. Uncertainty of description is more difficult to characterize, due to the lack of clarity of the (system) description. If unknown factors are missing in the description, the consequences cannot be measured (Weck and Eckert, 2007).

In my thesis I would like to focus on the uncertainty of measurement based on the model of Weck and Eckert (2007), since this type of uncertainty can be modeled well. In my research I examine the effect of the measurement uncertainty by the application of statistical control charts that are outstanding tools in production management. Therefore this work is directly related to the operations risk and customer risk based on the risk-categorization by the review of Harland et al. (2003).

Appendix B

Table of papers - control charts

TABLE B.1: Table of articles - Control Charts - 1 (U=univariate, M=multivatiate, P=parametric, NP=nonparametric, F=fixed, A=adaptive, T=traditional, R=risk-based)

Nr.	Article	Year	Dimension	Distribution	Chart Parameters	Type
1	Shewhart, 1931	1931	U	P	F	T
2	Shewhart and Deming, 1939	1939	U	P	F	T
3	Hotelling, 1947	1947	M	P	F	T
4	Page, 1954	1954	U	P	F	T
5	Duncan, 1956	1956	U	P	F	T
6	Jackson, 1959	1959	M	P	F	T
7	Zimmer, 1963	1963	U	NP	F	T
8	Duncan, 1971	1971	U	P	F	T
9	Saniga and Shirland, 1977	1977	U	P	F	T
10	Abraham, 1977	1977	U	P	F	R
11	Bakir and Reynolds, 1979	1979	U	NP	F	T
12	Lashkari and Rahim, 1982	1982	U	NP	F	T
13	Alt, 1982	1982	U	P	F	T
14	Rahlm, 1985	1985	U	NP	F	R
15	Crosier, 1986	1986	U	P	F	T
16	Kanazuka, 1986	1986	U	P	F	R
17	Tuprah and Ncube, 1987	1987	U	P	F	T
18	Reynolds et al., 1988	1988	U	P	A	T
19	Lucas and Saccucci, 1990	1990	U	P	F	T
20	Reynolds et al., 1990	1990	U	P	A	T
21	Hackl and Ledolter, 1991	1991	U	NP	F	T
22	Runger and Pignatiello, 1991	1991	U	NP	A	T
23	Lowry et al., 1992	1992	M	P	F	T
24	Saccucci et al., 1992	1992	M	NP	A	T
25	Hackl and Ledolter, 1992	1992	U	NP	F	T
26	Yourstone and Zimmer, 1992	1992	U	NP	F	T
27	Prabhu et al., 1993	1993	U	P	A	T
28	Costa, 1994	1994	U	P	A	T
29	Amin et al., 1995	1995	U	NP	F	T
30	Lowry and Montgomery, 1995	1995	M	P	F	T
31	Margavio et al., 1995	1995	U	P	F	T
32	Annadi et al., 1995	1995	U	P	A	T
33	Aparisi, 1996	1996	M	P	A	T
34	Costa, 1997	1997	U	P	A	T
35	Prabhu et al., 1997	1997	M	NP	F	T
36	Chou et al., 1998	1998	U	NP	F	T
37	Mittag and Stemann, 1998	1998	U	P	F	R
38	Tagaras, 1998	1998	U	P	A	T
39	Chou et al., 2000	2000	U	NP	F	T
40	Luceno and Puig-pey, 2000	2000	U	P	F	T
41	Aparisi and Haro, 2001	2001	M	P	A	T
42	Reynolds and Arnold, 2001	2001	U	NP	A	T
43	Chou et al., 2001	2001	U	NP	F	T
44	De Magalhães et al., 2001	2001	U	P	A	T

TABLE B.2: Table of articles - Control Charts - 2 (U=univariate, M=multivatiate, P=parametric, NP=nonparametric, F=fixed, A=adaptive, T=traditional, R=risk-based)

Nr.	Article	Year	Dimension	Distribution	Chart Parameters	Type
45	Nenes, 2011	2001	U	P	F	T
46	Calzada and Scariano, 2001	2001	U	NP	F	T
47	Linna and Woodall, 2001	2001	U	P	F	R
48	Linna et al., 2001	2001	M	P	F	R
49	Stemann and Weihs, 2001	2001	U	P	F	R
50	Chou et al., 2002	2002	M	P	F	T
51	Jones et al., 2004	2004	U	P	F	T
52	Chen, 2004	2004	U	P	F	T
53	Maravelakis et al., 2004	2004	U	NP	F	R
54	Reynolds and Kim, 2005	2005	M	P	F	T
55	Knoth, 2005	2005	U	P	F	T
56	Montgomery, 2005	2005	U/M	P	F	T
57	Lin and Chou, 2005	2005	U	NP	A	T
58	He and Grigoryan, 2006	2006	U	P	A	T
59	Bakir, 2006	2006	U	NP	F	T
60	Koutras et al., 2006	2006	M	P	F	T
61	Faraz and Parsian, 2006	2006	M	P	A	T
62	Chen and Hsieh, 2007	2007	M	P	A	T
63	Ferrer, 2007	2007	M	P	F	T
64	Bersimis et al., 2007	2007	U	P	F	T
65	Chen, 2007	2007	U	P	A	T
66	Kao and Ho, 2007	2007	U	NP	F	T
67	Chen and Cheng, 2007	2007	U	NP	F	T
68	Lin and Chou, 2007	2007	U	NP	A	T
69	Huwang and Hung, 2007	2007	M	P	F	R
70	Song and Vorburger, 2007	2007	U	P	A	T
71	Qiu, 2008	2008	M	NP	F	T
72	Serel and Moskowitz, 2008	2008	U	P	F	T
73	De Magalhães et al., 2009	2009	U	P	A	T
74	Das, 2009	2009	M	NP	F	T
75	Aparisi and Luna, 2009	2009	M	P	F	T
76	Wu et al., 2009	2009	U	NP	F	T
77	Luo et al., 2009	2009	U	P	A	T
78	Yang and Yu, 2009	2009	U	NP	F	T
79	Panagiotidou and Nenes, 2009	2009	U	P	A	T
80	Bush et al., 2010	2010	M	NP	F	T
81	Zhang et al., 2010	2010	M	P	F	T
82	Faraz et al., 2010	2010	M	P	A	T
83	Abbasi, 2010	2010	U	NP	F	R
84	Castagliola and Maravelakis, 2011	2011	U	P	F	T
85	Qiu and Li, 2011	2011	U	NP	F	T
86	Yang et al., 2011	2011	U	NP	F	T
87	Lee, 2011	2011	U	P	A	T
88	Reynolds and Cho, 2011	2011	M	P	A	T
89	Faraz and Saniga, 2011	2011	U	P	A	T
90	Tasias and Nenes, 2012	2012	U	P	A	T
91	Graham et al., 2012	2012	U	NP	F	T
92	Maravelakis, 2012	2012	U	P	F	R
93	Epprecht et al., 2013	2013	M	P	A	T
94	Lee, 2013	2013	M	P	A	T
95	Phaladiganon et al., 2013	2013	M	NP	F	T
96	Bashiri et al., 2013	2013	U	P	F	T
97	Hegedűs et al., 2013b	2013	U	NP	F	R
98	Tuerhong et al., 2014	2014	M	NP	F	T
99	Ganguly and Patel, 2014	2014	U	P	F	T
100	Chong et al., 2014	2014	U	P	A	T

TABLE B.3: Table of articles - Control Charts - 3 (U=univariate, M=multivatiate, P=parametric, NP=nonparametric, F=fixed, A=adaptive, T=traditional, R=risk-based)

Nr.	Article	Year	Dimension	Distribution	Chart Parameters	Type
101	Faraz et al., 2014	2014	M	P	A	T
102	Khurshid and Chakraborty, 2014	2014	U	NP	F	R
103	Riaz, 2014	2014	U	P	F	R
104	Zhang et al., 2014	2015	U	P	F	T
105	Abbasi, 2014	2015	U	P	F	R
106	Cheng and Shiau, 2015	2015	M	NP	F	T
107	Haq et al., 2015	2015	U	NP	F	R
108	Joekes et al., 2015	2015	U	P	A	T
109	Seif et al., 2015	2015	M	P	A	T
110	Chew et al., 2015	2015	M	P	A	T
111	Maleki et al., 2016	2016	U	P	F	R
112	Abbasi, 2016	2016	U	NP	F	R
113	Aslam et al., 2016	2016	U	P	A	T
114	Tran et al., 2016	2016	U	NP	F	R
115	Yeong et al., 2016	2016	M	P	F	T
116	Hu et al., 2016b	2016	U	P	A	R
117	Hu et al., 2016a	2016	U	P	A	R
118	Chen et al., 2016	2016	M	NP	F	T
119	Yue and Liu, 2017	2017	M	NP	A	T
120	Chattinnawat and Bilen, 2017	2017	M	P	F	R
121	Daryabari et al., 2017	2017	U	NP	F	R
122	Salmasnia et al., 2018	2018	M	P	A	T
123	Pawar et al., 2018	2018	U	NP	A	T
124	Safe et al., 2018	2018	U	P	A	T
125	Amiri et al., 2018	2018	M	P	F	R
126	Cheng and Wang, 2018	2018	U	P	F	R

Appendix C

Table of papers - Measurement uncertainty

TABLE C.1: Table of articles - Measurement Uncertainty - 1 (E=evaluation, C=conformity, S=symmetric, A=asymmetric)

Nr.	Article	Year	Topic	Distribution
1	BIPM et al., 1995	1995	E	S
2	ILAC, 1996	1996	E	S
3	Tsai and Johnson, 1998	1998	E	S
4	King, 1999	1999	C	S
5	Lira, 1999	1999	C	S
6	Currie, 2001	2001	E	A
7	Mauris et al., 2001	2001	E	S
8	AIAG, 2002	2002	E	S
9	Eurachem, 2002	2002	E	S
10	Martens, 2002	2002	E	A
11	ASME, 2002	2002	C	S
12	Lira, 2002	2002	C	S
13	EA, 2003	2003	E	S
14	ISO-TC69, 2003	2003	C	S
15	Källgren et al., 2003	2003	C	S
16	Choi et al., 2003a	2003	E	S
17	Choi et al., 2003b	2003	E	S
18	Kudryashova and Chunovkina, 2003	2003	E	S
19	Herrador and Gonzalez, 2004	2004	E	A
20	D'Agostini, 2004	2004	E	A
21	Ferrero and Salicone, 2004	2004	E	S
22	Herrador et al., 2005	2005	E	A
23	Douglas et al., 2005	2005	E	A
24	Desimoni and Brunetti, 2005	2005	C	S
25	Willink, 2005	2005	E	A
26	Cordero and Roth, 2005	2005	E	S
27	Pendrill and Källgren, 2006	2006	C	S
28	Forbes, 2006	2006	C	S
29	Désenfant and Priel, 2006	2005	E	S
30	Cowen and Ellison, 2006	2006	E	A
31	Synek, 2006	2006	E	A
32	Rossi and Crenna, 2006	2006	C	A
33	Desimoni and Brunetti, 2006	2006	C	S
34	Willink, 2006	2006	E	A
35	Pendrill, 2006	2006	C	A
36	Hinrichs, 2006	2006	C	S
37	Bich et al., 2006	2006	E	S
38	Lampasi et al., 2006	2006	E	S
39	**Eurachem2007**	2007	C	S
40	Pavese, 2007	2007	E	S

TABLE C.2: Table of articles - Measurement Uncertainty - 2 (E=evaluation, C=conformity, S=symmetric, A=asymmetric)

Nr.	Article	Year	Topic	Distribution
41	Pendrill, 2007	2007	C	S
42	Macii and Petri, 2007	2007	E	S
43	Synek, 2007	2007	E	A
44	BIPM et al., 2008	2008	E	S
45	Mekid and Vaja, 2008	2008	E	A
46	Pendrill, 2008	2008	C	S
47	Williams, 2008	2008	C	A
48	Sim and Lim, 2008	2008	E	A
49	Richardson et al., 2008	2008	E	A
50	Pavese, 2009	2009	E	S
51	Pavlovcic et al., 2009	2009	E	A
52	Pendrill, 2009	2009	C	A
53	Macii and Petri, 2009	2009	C	S
54	Sommer, 2009	2009	E	A
55	Vilbaste et al., 2010	2010	E	A
56	Hinrichs, 2010	2010	C	S
57	Pendrill, 2010	2010	C	S
58	Beges et al., 2010	2010	C	S
59	Lira and Grientschnig, 2010	2010	E	S
60	Shainyak, 2013	2013	C	S
61	Boumans, 2013	2013	E	S
62	Benoit, 2013	2013	E	S
63	Possolo, 2013	2013	E	S
64	Pendrill, 2014	2014	C	A
65	Huang, 2014	2014	C	S
66	Theodorou and Zannikos, 2014	2014	C	S
67	Fernández et al., 2014	2014	E	S
68	Koshulyan and Malaychuk, 2014	2014	E	S
69	Bich, 2014	2014	E	S
70	Huang, 2015	2015	C	S
71	Volodarsky et al., 2015	2015	C	S
72	Ramsey and Ellison, 2015	2015	E	A
73	Wiora et al., 2016	2016	E	S
74	Rajan et al., 2016a	2016	E	A
75	Rajan et al., 2016b	2016	E	A
76	Fabricio et al., 2016	2016	E	S
77	Lira, 2016	2016	E	S
78	Eurolab, 2017	2017	C	S
79	Herndon, 2017	2017	E	S
80	Molognoni et al., 2017	2017	C	A
81	Kuselman et al., 2017a	2017	C	S
82	Kuselman et al., 2017b	2017	C	S
83	Dastmardi et al., 2018	2018	C	S
84	Pennecchi et al., 2018	2018	C	S
85	Possolo and Bodnar, 2018	2018	E	S
86	Wang et al., 2018	2018	E	S

Appendix D

Reviewed studies including the consideration of measurement errors

TABLE D.1: Elements of the cost of the decision outcomes

Reference	**Univariate/ Multivariate**	**Parametric/ Nonparametric**	**Adaptive/ Fixed parameters**	**Control chart**
Abraham, 1977	Univariate	Parametric	Fixed parameters	$\overline{X}$
Rahlm, 1985	Univariate	Non-parametric	Fixed parameters	$\overline{X}$
Kanazuka, 1986	Univariate	Parametric	Fixed parameters	$\overline{X}-S$
Mittag and Stemann, 1998	Univariate	Parametric	Fixed parameters	$\overline{X}-R$
Stemann and Weihs, 2001	Univariate	Parametric	Fixed parameters	$\overline{X}-S$, EWMA
Linna et al., 2001	Multivariate	Parametric	Fixed parameters	χ^2
Linna and Woodall, 2001	Univariate	Parametric	Fixed parameters	$\overline{X}-S^2$
Maravelakis et al., 2004	Univariate	Non-Parametric	Fixed parameters	EWMA
Huwang and Hung, 2007	Multivariate	Parametric	Fixed parameters	|S|
Abbasi, 2010	Univariate	Non-parametric	Fixed parameters	EWMA
Maravelakis, 2012	Univariate	Parametric	Fixed parameters	CUSUM
Hegedűs et al., 2013b	Univariate	Parametric/ Non-parametric	Fixed parameters	$\overline{X}$ MA, EWMA
Katona et al., 2014	Univariate	Non-parametric	Fixed parameters	EWMA
Abbasi, 2014	Univariate	Parametric/ Non-parametric	Fixed parameters	*Shewhart* CUSUM, EWMA
Riaz, 2014	Univariate	Parametric	Fixed parameters	$\overline{X}, S, S^2$
Haq et al., 2015	Univariate	Non-parametric	Fixed parameters	EWMA
Hu et al., 2015	Univariate	Parametric	Fixed parameters	synthetic $\overline{X}$
Abbasi, 2016	Univariate	Non-parametric	Fixed parameters	EWMA
Maleki et al., 2016	Multivariate	Parametric	Fixed parameters	ELR
Tran et al., 2016	Multivariate	Parametric	Fixed parameters	Shewhart-RZ
Hu et al., 2016a	Univariate	Parametric	Adaptive (VSS)	VSS $\overline{X}$
Hu et al., 2016b	Univariate	Parametric	Adaptive (VSI)	VSI $\overline{X}$
Chattinnawat and Bilen, 2017	Multivariate	Parametric	Fixed parameters	T^2
Daryabari et al., 2017	Univariate	Parametric	Fixed parameters	MAX EWMAMS
Cheng and Wang, 2018	Univariate	Parametric	Fixed parameters	CUSUM median EWMA median
Amiri et al., 2018	Multivariate	Parametric	Fixed parameters	GLR, MEWMA

Appendix E

Description of the adaptive control chart rules

Consider a process with observed values following a normal distribution with expected value μ and variance σ^2. When FP control chart (control chart with fixed parameters) is used to monitor the aforementioned process, a random sample (n_0) is taken every hour (denoted by h_0).

In the case of a VSSI control chart, two different levels can be determined for the contol chart parameters (n, h). The first level represents a parameter set with loose control (n_1, h_1) including smaller sample size and longer sampling interval, and the second level means a strict control policy (n_2, h_2) with a larger sample size and shorter sampling interval. Nevertheless, n and h must satisfy the following relations: $n_1 < n_0 < n_2$ and $h_2 < h_0 < h_1$, where n_0 is the sample size and h_0 is the sampling interval of the FP control chart. The switch rule between the parameter levels is determined by a warning limit coefficient w indicating the specification of central and warning regions (Chen et al., 2007):

$$I_1(i) = \left[\frac{\mu_0 - w\sigma}{\sqrt{n(i)}}, \frac{\mu_0 + w\sigma}{\sqrt{n(i)}}\right] \tag{E.1}$$

and

$$I_2(i) = \left[\frac{\mu_0 - k\sigma}{\sqrt{n(i)}}, \frac{\mu_0 - w\sigma}{\sqrt{n(i)}}\right] \cup \left[\frac{\mu_0 + w\sigma}{\sqrt{n(i)}}, \frac{\mu_0 + k\sigma}{\sqrt{n(i)}}\right] \tag{E.2}$$

$$I_3(i) = \overline{I_1 \cup I_2} \tag{E.3}$$

where $i = 1, 2...$ is the number of the sample, I_1denotes the central region, and I_2 the warning region. During the control process, the following decisions can be made (Lim et al., 2015):

1. If $\overline{x}_i \in I_1$, the manufacturing process is in "in-control" state. Sample size n_1 and sampling interval h_1 are used to compute $\overline{x}_{i+1}$.

2. If $\overline{x}_i \in I_2$, the monitored process is "in-control" but $\overline{x}_i$ falls in the warning region; thus, n_2 and h_2 are used for the $(i+1)^{th}$ sample.

3. If $\overline{x}_i \notin I_1$ and $\overline{x}_i \notin I_2$, the process is out of control, and corrective actions must be taken. After the corrective action, $\overline{x}_{i+1}$ falls into the central region (assuming that the correction was successful), but there is no previous sample to determine $n(i+1)$ and $h(i+1)$. Therefore, as Prabhu et al. (1994) and Costa (1994) proposed, the next sample size and interval are selected randomly with probability p_0. p_0 denotes the probability that the sample mean falls within

the central region. Similarly, $1 - p_0$ is the probability, meaning that the sample point falls within the warning region.

Appendix F

Examples for measurement process monitoring techniques

MSA Handbook (Measurement System Analysis) determines five categories of measurement system error: bias, repeatability, reproducibility, stability and linearity (AIAG, 2010). Different statistical methods can be used to assess measurement system performance considering the five aforementioned categories. This section clarifies the meaning of the categories and discusses the suggested methods for analysis.

Bias

Bias is the difference between the reference value and the detected average of multiple measurements when considering the same characteristic of the same part (Pyzdek, 2003, AIAG, 2010). Bias can be determined through experimental measurements using a certified etalon. Detailed guidance and practical examples are provided by Shaji, 2006, Sibalija and Majstorovic (2007), AIAG (2010), Sahay (2010), Yu (2012).

Stability

Stability is the change in bias over an extended time period. It is the variation of the measurement result when same characteristic is measured on the same part (by the same person) over a time period. Stability can be analyzed by $\overline{\mathrm{X}}$-R charts, for practical example, see Shaji (2006), Sibalija and Majstorovic (2007), Sahay (2010), Pai et al. (2015).

Linearity

Similarly to stability, linearity is associated with the examination of bias however, linearity refers to the bias throughout the expected operating range. AIAG (2010) suggests to use at least five parts for the experiment that cover the operating range of the examined gage. Each part should be measured at least ten times and average bias values must be calculated against the reference values. Linear fitting can be conducted if average bias values are plotted with respect to the reference values:

$$\overline{bias_i} = ax_i + b \tag{F.1}$$

where $\overline{bias_i}$ is the bias average and x_i is the reference value, a is the slope and b is the intercept of the fitted line. Gage linearity is acceptable if "bias=0" line is located entirely within the confidence bounds of the fitted curve. Detailed numerical examples are provided by Shaji (2006), Sibalija and Majstorovic (2007), AIAG (2010), Sahay (2010), Yu (2012), Pai et al. (2015), Mat-Shayuti and Adzhar (2017).

Repeatability and reproducibility

AIAG (2010) refers to repeatability as "within apparaiser" variability. In other words, it is the variation in measurements when a single characteristic is measured several times on the same part by the same appraiser and using the same device.

Despite the repeatability, reproducibility aims to characterize variation "between appraisers". In this case a single characteristic is measured several times on the same part using the same device but the measurement is conducted by different appraisers (AIAG, 2010). Gage R&R (or GRR) is the proposed method to analyze the variation regarding repeatability and reproducibility. The study can be conducted based on different approaches:

- Range method
- Average and Range method
- ANOVA method

Range method

This is an approximation of measurement variability. Usually two appraisers participates in the study who measure the same part (5 parts) once with the same instrument. Range method does not decompose variability into repeatability and reproducibility, it focuses on the ratio of average range of obtained measurements and process standard deviation:

$$GRR = \frac{\overline{R}}{d_2} = \frac{1}{d_2}\frac{\sum R_i}{n} \tag{F.2}$$

where R_i is the range of the obtained measurements by appraiser "A" and "B" regarding part i and d_2 is the correction constant. Based on F.2, the result can be expressed related to the process variation:

$$\%GRR = 100 * \left(\frac{GRR}{\text{Process Standard Deviation}}\right) \tag{F.3}$$

For details and practical examples, see AIAG (2010), Sahay (2010).

Average and Range method

Despite Range method, this approach provides information about repeatability and reproducibility. Three appraisers are recommended to participate in the study. They need to measure at least ten parts, each part is measured three times by each appraiser (without seeing each others' results) (AIAG, 2010). In this case the GRR value can be expressed by equipment variation (repeatability) and appraiser variation (reproducibility):

$$GRR = \sqrt{EV^2 + AV^2} \tag{F.4}$$

where EV is the equipment variation and AV is the appraiser variation respectively. GRR can be also represented relative to the total variation (TV):

$$\%GRR = 100 * \left(\frac{GRR}{TV}\right) \tag{F.5}$$

Average and Range method was applied by several scholars to investigate measurement system repeatability and reproducibility: Mohamed and Davahran (2006), Sibalija and Majstorovic (2007), AIAG (2010), Sahay (2010), Dalalah and Diabat (2015), Mat-Shayuti and Adzhar (2017).

ANOVA method

Analysis Of Variance method provides more information than Average and Range method, since it is also able to characterize the interaction between parts and appraisers. The data collection procedure is the same as it is described by Average and Range method however, ANOVA table is used in order to decompose the variance into specific components: parts, appraisers, interaction between appraisers and parts, and finally, repeatability due to the measurement device (AIAG, 2010). As outcome, the components' contribution to total variance can be expressed:

$$\%Contribution = 100 * \left(\frac{\sigma^2_{(components)}}{\sigma^2_{(total)}} \right) \tag{F.6}$$

Numerical examples are provided by Senol (2004), Mohamed and Davahran (2006), Sibalija and Majstorovic (2007), Kazerouni (2009), AIAG (2010), Mat-Shayuti and Adzhar (2017).

Appendix G

The author's publications related to the topic

International Journal Articles

Kosztyán, Z. T., & **Katona, A. I.** (2018). Risk-Based X-bar chart with variable sample size and sampling interval. In: *Computers & Industrial Engineering*, 120, 308-319.

Kosztyán, Zs. T., Hegedűs, Cs. and **Katona, A. I.** (2017). Treating measurement uncertainty in industrial conformity control. In: *Central European Journal of Operations Research*, pp. 1-22. ISSN: 1613-9178.

Kosztyán, Z. T., & **Katona, A. I.** (2016). Risk-based multivariate control chart. In: *Expert Systems with Applications*, 62, 250-262.

Cs. Hegedűs, **A. Katona**, Zs. T Kosztyán (2014): Design and Selection of Risk-Based Control Charts, *Global Journal on Technology 5: pp. 92-98. 4th World Conference on Information Technology (WCIT-2013)*. Brussels, Belgium: 2013.11.26 -2013.11.28.

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Hungarian Articles

Katona A. I. (2015): Kockázatalapú többváltozós szabályozókártya kidolgozása a Le Bélier Magyarország Formaöntöde Zrt.-nél, *Logisztikai Híradó*, 25(1), pp. 15-18.

Kosztyán Zs T, **Katona A I** (2014): Kockázatalapú változó paraméterű szabályozó kártya kidolgozása a statisztikai folyamatszabályozásban, *Taylor: Gazdálkodás- és Szervezéstudományi Folyóirat: A Virtuális Intézet Közép-Európa Kutatására Közleményei 6*,pp. 16-17.

Kosztyán Zs. T., **Katona A.**, Hegedűs Cs. (2014): Új kockázatalapú szabályozó kártyák tervezése, kiválasztása és folyamathoz illesztése, *Taylor: Gazdálkodás- és Szervezéstudományi Folyóirat: A Virtuális Intézet Közép-Európa Kutatására Közleményei 6:(3-4)*, pp. 188-195.

Katona A. I. (2013): A beavatkozási határok módosítása a mérési bizonytalanság, valamint a termékparaméterek megváltozásának figyelembevételével a statisztikai folyamatszabályozásban, *E-Conom*, 2(2), pp. 35-45.

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Katona A. I. (2012): A statisztikai folyamatszabályozás bevezetése, *Logisztikai Híradó*, 23(1), pp. 34-37.

Book Chapters

Katona A. I. (2015): Ellenőrző kártya-illesztési folyamat kidolgozása a mérési bizonytalanság figyelembevételével a statisztikai folyamatszabályozásban. *Tudós Bagoly Válogatás a XXXI. Országos Tudományos Diákköri Konferencia Közgazdaságtudományi Szekciójában bemutatott díjnyertes dolgozatokból*, pp. 431-489.

Proceedings

Kosztyán Zs. T., **Katona A.** (2014): Kockázatalapú többváltozós szabályozó kártya kidolgozása a mérési bizonytalanság figyelembevételével, *Kulturális és társadalmi sokszínűség a változó gazdasági környezetben: 2. IRI Társadalomtudományi Konferencia, Nové Zámky, Szlovákia*, 2014.04.25-2014.04.26. Komárno: International Research Institute, pp. 151-164.

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