PhD dissertation

NOVEL PROCESS GRAPH-BASED SOLUTIONS OF INDUSTRY 4.0 FOCUSED OPTIMISATION PROBLEMS

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Contents

Abstract

As most of the energy production and transformation processes are safety-critical, it is vital to develop tools that support the analysis and minimisation of their reliability-related risks. The resultant optimisation problem should reflect the structure of the process which requires the utilisation of flexible and problem-relevant models.

Process graphs (P-graphs) have been proven to be useful in identifying optimal structures of process systems and business processes. The provision of redundant critical units can significantly reduce operational risk. Redundant units and subsystems can be modelled in P-graphs by adding nodes that represent logical conditions of the operation of the units. In Chapter 2 it is revealed that P-graphs extended by logical condition units can be transformed into reliability block diagrams, and based on the cut sets and path sets of the graph a polynomial risk model can be extracted. Since the exponents of the polynomial represent the number of redundant units, the cost function of the reliability redundancy allocation problem as a nonlinear integer programming model can be formalised, where the cost function handles the costs associated with consequences of equipment failure and repair times. The applicability of this approach presented in Chapter 3 is illustrated in a case study related to the asset-intensive chemical, oil, gas and energy sector. The results show that the proposed algorithm is useful for risk-based priority resource allocation in a reforming reaction system.

Chapter 4 of my dissertation deals with the optimisation of quality assurance processes. Testing is an indispensable process for ensuring product quality in production systems. Reducing the time and cost

spent on testing whilst minimising the risk of not detecting faults is an essential problem of process engineering. The optimisation of complex testing processes consisting of independent test steps is considered. Survival analysis based models of an elementary test to efficiently combine the time-dependent outcome of the tests and costs related to the operation of the testing system are developed. A mixed integer nonlinear programming (MINLP) model to formalize how the total cost of testing depends on the sequence and the parameters of the elementary test steps is proposed.

To provide an efficient formalization of the scheduling problem and avoid difficulties due to the relaxation of the integer variables, the MINLP model as a P-graph representation-based process network synthesis problem is considered. The applicability of the methodology is demonstrated by a realistic case study taken from the computer manufacturing industry. With the application of the optimal test times and sequence provided by the SCIP (Solving Constraint Integer Programs) solver, 0.1-5% of the cost of the testing can be saved.

Chapter 1

Introduction

1.1 Optimisation issues of the Industry 4.0 concept

The world is increasingly struggling with traditional manufacturing trends and ever-evolving digitalization. Companies' manufacturing processes need to change very rapidly to keep pace with competitors in their sectors. Digital technology transforms manufacturing in a big way nowadays. The fourth industrial revolution means that machines and production systems are connected to the information network, which are also integrated into an intelligent information system. Industrial revolutions have always radically changed production and manufacturing. The fourth industrial revolution is first and foremost a revolution of efficiency. With the emergence of new technologies, such as artificial intelligence, smart devices, and IoT tools, the main objective has become to integrate the latest achievements in IT with the previous achievements of the industrial revolution, thus forming a whole. Since the 1950s, the variety of products has been steadily increasing. Custom manufacturing and regionalization have become key issues. Also at the end of the 1950s, mass production reached its peak, and since then fewer and fewer versions of a particular product have been sold. For consumers, uniqueness has become an increasingly important value [21].

According to experts, digitalization will change production management at a fundamental level to develop production capacity that grows in line with customer demand and accelerate logistics services. Technological advances will also reform supply chain management. The spread of Industry 4.0 will lead to a larger product portfolio and an increased need for capacity. Related to manufacturing, there will be a need for greater speed and increasing demand for quality products. Industry 4.0 has brought and will continue to bring significant changes to the small and medium-sized enterprises (SME), depending on how well prepared SMEs are and how well they are able to keep pace with large enterprises in terms of technological innovation. They will either acquire new markets or, due to a lack of capital and knowledge, their infrastructural and economic backwardness will increase and they will not be able to integrate.

When smart manufacturing systems, which link smart factories and smart products in series, are combined with smart logistics processes, it encompasses a complex manufacturing process, together with marketing activities and smart services. Thus, a strong customer-centric and demand-oriented production management can be created, meeting specific needs. This is vertical integration, in which, however, there is a strong focus on how real-world elements are represented in a system that can be interpreted by information systems. This type of integration describes five levels [98] by the ISA-95 standard [85]. At the lowest level 0 is the actual production phase that is implemented. The first and second levels are where the process is manipulated, controlled, and changes are made. The third level is where process analysis, detailed scheduling, reliability issues, and continuous availability are performed. At the fourth level, enterprise logic, planning, and logistics are implemented (Fig. 1.1).

This is also where decisions are made on the number of raw materials. Horizontal integration follows the product throughout its life cycle. Starting with the design, through sourcing of raw materials and manufacturing, to delivery. A tool is therefore needed that can manage and optimize these processes in parallel. Both suppliers and customers are part of the chain [11].

The four pillars of Industry 4.0 are interconnectivity, decentralized decision-making,

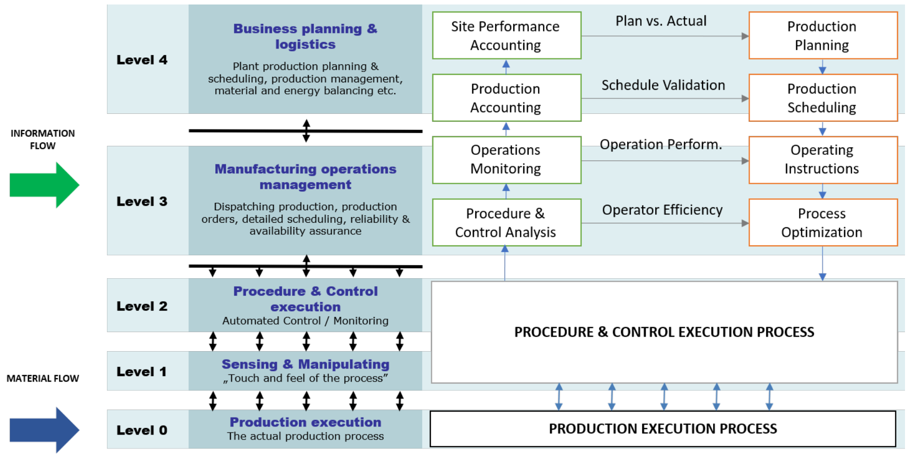

Figure 1.1: The structure of the ISA-95 standard for constructing vertical integration [20]

information transparency, and technical backup. However, these are not enough on their own and additional elements need to be introduced. Examples include smart factories, smart manufacturing, product customization, agility, autonomy, modularity, and flexibility [50]. Of these, smart factories, virtualization, flexibility, sustainability, and real-time availability have the greatest impact on optimisation. Digital twins have a key role to play in Industry 4.0 [80]. A digital twin is a computer program that uses real-world data to create simulations that calculate the operation and performance of a product or process. It requires data from a real-life object to develop a model that can simulate the original item. The resulting virtual replica of the physical object can provide feedback on the original version using a variety of sensors and data collection devices. Industry 4.0 is the most complex model ever. A lot of data is being fed into large databases every day. Processing it is a big challenge, which is where big data systems come in. Previous database management tools would have faced considerable difficulties in solving these problems, as the type of data is not only large in number but also arriving at high speeds and is very diverse. Another critical element is decentralization to ensure that problems are addressed at the right place/level. In addition, Industry 4.0 is also committed to the idea that if a problem can be broken down into subproblems, then let's do it. Particular attention needs to be paid to overlapping problems that are located between layers. Furthermore, the issue of sustainability has become critical [20].

With the increase in production, there is a need for rapid changeover of production lines to enable the manufacture of small, one-off products. In the past, it was only economical to mass-produce a large number of items in series, but new designs can now meet the ever faster-changing needs of individual customers (Fig. 1.2).

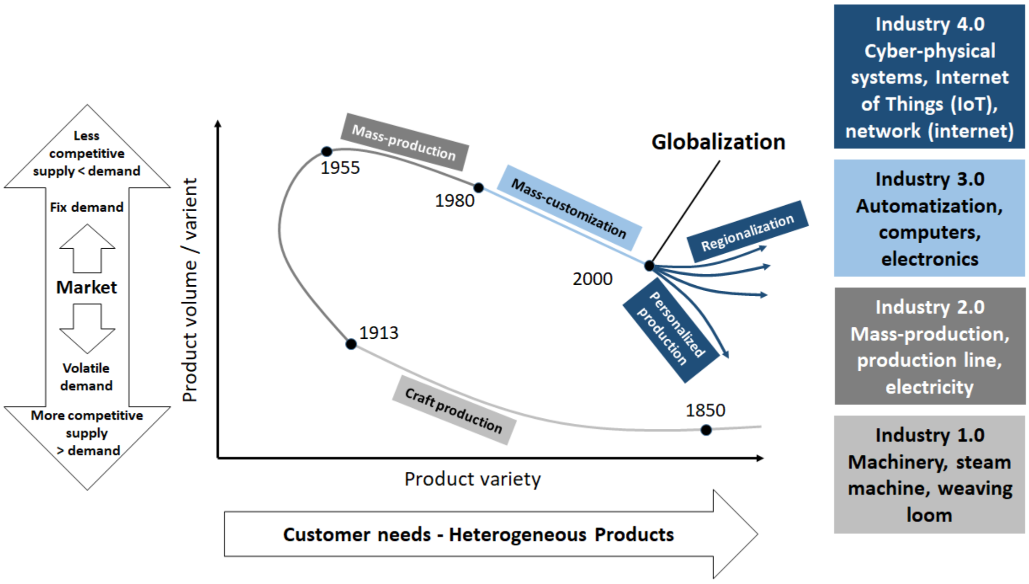

Figure 1.2: The evolution of industry and the change in drivers [20]

By analyzing these processes to optimize production, it is possible to eliminate the production of rejects and reduce costs. In addition, the positive environmental impact of Industry 4.0 can be a positive factor. With new and green technologies, production will produce fewer defective products, thus reducing the number of rejects, which will reduce waste. There will be companies that will not be able to keep up with digitalization, which will not only cause problems for that company, but also for other companies further down the supply chain, as they may or may not be integrated into the supply chains, and their exit will reorganize the supply chains and may lead to a reorganization of the supply chains. The primary barriers to digital technology connectivity are:

- huge investment costs,

- lack or inappropriate allocation of resources,
- or increasingly stringent regulatory compliance.

Businesses without sufficient capital cannot make these investments on their own. For small and medium-sized enterprises, only state aid is likely to help, as they face financial, technological, and human resource challenges that are more difficult to overcome than multinationals.

The changes that have been initiated so far will continue to evolve in the coming period, providing opportunities to optimize complex manufacturing processes through simulation. This requires production data from sensors. Production models that are trained using data from real production can be repeated until the production process outlined is the most efficient. The data, software, and network technologies, the operation, management, and optimisation of the manufacturing facilities, allow for the consideration of individual needs in mass production and the optimisation of processes. Production lines can be designed to adapt to both current orders and specific customer needs. In addition, data collected by the company's order processing systems can be transmitted to workstations in the manufacturing facility. In the future, the manufacturing process can be automatically adapted to the urgency of incoming orders in the production network, from order receipt to material and tool ordering, through to production and delivery. This will enable adaptive manufacturing. In this system, orders will be processed in a process-coupled manner and can be passed on to the purchasing and logistics systems. The network linking the departments will ensure the optimal flow of materials and energy along the entire value chain. In such production processes, it is now possible to know exactly which parts are needed wherein a given operation and which machining operation will be next. The system checks the quality standards to be met by the finished product and analyses where potential bottlenecks in the process are based on the defects detected. To make this process work, machines, handling equipment, and warehouses or warehouse gates communicate independently with each other via networks using many sensors. In such a set-up, the design of the manufacturing environment needs to identify potential existing sources of information, process them and then model them before going live. This will require a combination of technologies that can filter, analyze and integrate data from different sources with existing IT systems.

In smart factories with cyber-physical systems [50, 28], real operational processes are mapped into a virtual world where production processes can be monitored and interactive intervention points can be created, enabling production optimisation and automated decision-making. Networked production will ensure a continuous flow of information and optimized production by managing data from various sensors, and will be linked to the intelligent workpiece, which will also tell the machine how to machine using a built-in sensor. To do this, each workpiece will be equipped with a digital identifier (e.g. RFID) containing all specifications and production parameters. The five essential elements of networked manufacturing:

- digital workpieces,
- intelligent machine,
- vertical networking,
- horizontal networking,
- smart workpiece.

Unconstrained problems are the simplest, and the easiest to solve, but they are the rarest. The next in terms of complexity is the so-called linear problem, where both the objective function and the constraints are linear. This is much more common and, in general, a significant proportion of the more complex optimisation problems can be decomposed into linear problems. A typical example is when one needs to calculate how much of a given product to produce with different constraints on the raw materials. More complex is the so-called quadratic programming. Here the constraints are still linear, but the objective function is complicated. More serious is the problem where both the constraints and the objective function are non-linear. These are called non-linear models or NLP, such as the calculation of preventive maintenance, taking into account energy efficiency. If our variables are integers, we are talking about integer programming. If the work to be done has to be paired with the workers, that is a good example of this kind of problem. And if you have a mixture of integers and continuous variables, that's MIP (Mixed-Integer Programming). In practice, a company may have more than one objective. For example, to maximize revenue and minimize the number of workers. If there is more than one objective function, it is called a multi-objective task. Finally, we may also need to introduce certain probabilities, stochastic elements, into the model. Risk tolerance and uncertainty factors associated with different elements may be introduced into the system. This further complicates the mathematical model and stochastic problems are typically solved in a two-stage system. In addition, there is the issue of so-called fuzzy programming. The main difference with respect to the former is that here the objectives, constraints and various parameters are ambiguous. There is also stochastic dynamic programming, in which the problem is built up from subproblems, and iteratively, as each state is run, it influences the next state. It is important to note that in the industrial environment we do not always need the exact outcome, often a good approximation can be sufficient, it is worth going in this direction if the model is too complex.

Size, modularity, complexity, adaptability, and quality of the solution are the most important properties of an optimisation model [22]. Size describes the size of the task, and how many variables and constraints there are. Modularity describes the extent to which the task can be decomposed, often individual components can be solved in parallel, which can significantly speed up the solution time of the overall task. And complexity can mean many things. It can mean static complexity, dynamic complexity, detail, and manufacturing complexity. The most important is mathematical or computational complexity. This is where the step size of the algorithm becomes important, what is the time step of the algorithm. The two most important categories are NP-hard and non NP-hard problems. Problems in the former category can often not be solved in the real environment (where there are many input parameters) in a reasonable time. Here also exist additional categories like factorial or exponential. The quality of the solution is how accurate the solution is, and how close it is to reality. An exact solution often is not required if it would take a too long time to calculate it, a good approximation may be sufficient. In general, the more accurate or precise the model, the more accurate the solution.

Among the areas of Industry 4.0, optimisation tasks are located at the fourth level of the ISA model. These tasks include maintenance management, ensuring reliable system operation, solutions to reduce the human resources used, streamlining quality assurance processes, and various scheduling tasks. In my dissertation, I investigate the optimisation of these areas and provide novel models and algorithms for their efficient management. The focus of the investigations is on the so-called P-graph based problem representation, so the following subsection describes the most important basic concepts of this area.

1.2 Fundamentals of the P-graph methodology

The goal of a process network synthesis is to create products from raw materials through various transformations (e.g., activities, physical reactions, etc.). Several methods have been developed for handling process network synthesis tasks, since the combinatorial complexity of these problems can grow rapidly resulting in the calculations more difficult. An efficient approach based on P-graphs (process graphs) has been published by Friedler et al. in the early 1990s as an algorithmic aid for delineating the structures of PNS problems [38]. A PNS problem is defined by the products, raw materials and intermediate materials, as well as the operating units with the parameters (maximal amount of the raw materials, required amounts of the products, capacity, fix and proportional costs of the operating units, etc.). Two types of nodes are depicted in a P-graph: nodes for materials by circle, and nodes for operating units by rectangle.

For a set M of entities, a PNS problem can be given as a (P, R, O) triplet, where $P \subseteq M$ and $R \subseteq M$ are special material sets for product and raw type materials,

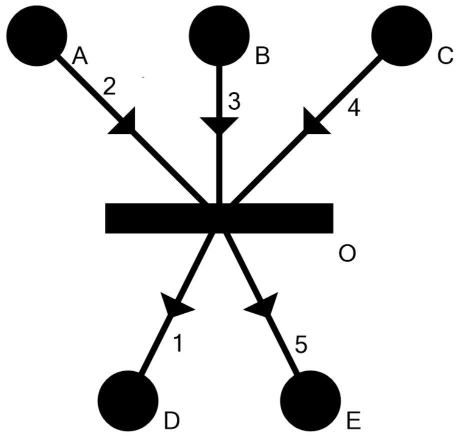

Figure 1.3: A simple P-graph representation

while $O \subseteq \wp(M) \times \wp(M)$ is the set of the operating units. Fig. 1.3 represents a simple P-graph with one operating unit ($O = (\{A, B, C\}, \{D, E\})$), three input ($A$, B, and C) and two output nodes (D, E). This illustrative example converts two parts of A, three parts of B, and four parts of C into one part of D and five parts of E. The constant ratios can be replaced by any functions which allow to describe more complex transformation steps between input and output quantities of the materials.

Although the process-network synthesis problem involves a mathematically difficult family of optimisation problems, the P-graph-based methodology and related algorithms can handle it efficiently. While the SSG (Solution Structure Generator) algorithm has been developed to automatically generate the possible solution structures [39], the ABB (Accelerated Branch-and-Bound) algorithm has been published to efficiently identify the best or n-best solution in terms of cost [36]. P-graph-based solutions have been applied in recent years in many areas, including optimal workflow generation, supply chain optimisation, and even efficient management of product supply problems [37].

1.3 Mathematical model of cost based synthesis problems using P-graphs

Creating the optimal structure of a system based on processes is called process synthesis. In practice, the problem definition used in process synthesis includes the definition of the available raw materials, the possible equipment (operating units), the products to be produced and the associated price, cost and constraint parameters. The objective of the optimisation task is to obtain the desired product using the raw materials and possible operating units so that the given objective function is minimised/maximised while considering all fixed constraints.

The purpose of this subsection is to present a mathematical model that describes a general process network synthesis problem. Several aspects can be given that can be the goal of the optimisation, so it can be about the shortest execution time, finding the most reliable process, but also cost/profit minimisation/maximisation can be crucial.

In the following, l present a mixed-integer linear programming model that can determine the optimal solution structure for manufacturing products from raw materials using operating units where profit maximisation is the objective function. First, let us see the general notations:

Let M be the set of materials $M = R \cup I \cup P$, where R is the set of raw materials, I is the set of intermediate materials, and P is the set of products. Let also denote the set O of the operating units and use the following two sets to define precisely the constraints:

$\varphi^-(m)$: the set of operating units that can produce material m

$\varphi^+(m)$: the set of operating units that use m as their input

The following parameters are introduced for the operating units:

For each $o \in O$:

- c_o: capacity value,

- a fix_o: fixed cost, and
- a $prop_o$: proportional cost.

For each $r \in R$ known:

- $price_r$: the price of the given raw material r, and
- an upper bound max_r represents that which is the available quantity of the present raw material.

For each intermediate material $i \in I$, we can specify

- its $price_i$ selling price;
- a max_i upper bound that controls the amount of material that can be retained in the considered material point;
- a $penal_i$ penalty value, which defines the penalty rate that appears as a cost if all the quantities in the considered material point are not consumed by an operating unit.

For each $p \in P$ there may exist

- a $price_p$ parameter that gives the revenue received per unit of product sold;
- a min_p which sets a lower bound on the quantity of product to be produced;
- a max_p value, which gives an upper bound on the market demand.

For the material balance, the following parameters are given: For each $o \in O$ and $m \in M$

- $ir_{o,m}$ is the ratio of the material m to the input of the operating unit o;
- $or_{o,m}$ is the ratio of the material m to the output of the operating unit o.

The decision variables of the mathematical model are the following: for each $o \in O$ there is a continuous variable $x_o \in R_0^+$ and an existence variable $y_o \in \{0, 1\}$. These represent the amount of material flowing through the operating unit, as well as, the state of the operating unit, depending on whether the unit is involved in a given structure or not. The constraints of the model are as follows:

For each operating unit ($\forall o \in O$):

$$x_o \leq M_0 \cdot y_o, \tag{1.1}$$

where M_0 represents the „Big-M" (upper bound of the capacity of the operating unit o).

For each raw material ($\forall r \in R$):

$$\sum_{o \in \varphi^+(r)} ir_{o,r} \cdot x_o \leq max_r \tag{1.2}$$

For all intermediate materials ($\forall i \in I$):

$$\sum_{o \in \varphi^-(i)} or_{o,i} \cdot x_o - \sum_{o \in \varphi^+(i)} ir_{o,i} \cdot x_o \leq max_i \tag{1.3}$$

For each product ($\forall p \in P$):

$$min_p \leq \sum_{o \in \varphi^-(p)} or_{o,p} \cdot x_o - \sum_{o \in \varphi^+(p)} ir_{o,p} \cdot x_o \leq max_p \tag{1.4}$$

As indicated above, the objective in general is to maximize profit, so the objective function can be written in the following form:

$$\begin{aligned} z = & \sum_{p \in P} (price_p(\sum_{o \in \varphi^-(p)} or_{o,p} \cdot x_o - \sum_{o \in \varphi^+(p)} ir_{o,p} \cdot x_o))+ \\ & \sum_{i \in I} ((price_i - penal_i)(\sum_{o \in \varphi^-(i)} or_{o,i} \cdot x_o - \sum_{o \in \varphi^+(i)} ir_{o,i} \cdot x_o))- \\ & \sum_{r \in R} (price_r \cdot \sum_{o \in \varphi^+(r)} ir_{o,r} \cdot x_o) - \sum_{o \in O} fix_o \cdot y_o + prop_o \cdot x_o \end{aligned} \tag{1.5}$$

The expression above includes the revenue from the sales of the products, the profit from the sales of intermediate materials and the difference between the penalties imposed, and the cost items arising from the purchase of raw materials and use of products. An important factor is also the weight of fixed and proportional costs which may result from the operation of the operating units.

1.4 Time-constrained process network synthesis

The lifecycle of a process lasts until the predefined goal or activity is achieved. Obviously, the aim is to finish the task as quickly as possible so that we can move on to the next phase as quickly as possible. A production process can be well defined using the process network synthesis method. In the previous sections, I introduced the analogy between the general producing processes and the P-graphs. If time also needs to be considered, the established methodology needs to be extended with time variables in the model [54]. The availability of resources can be limited, and all end states can be demanded to be reached. To obtain a suitable mathematical model, some other parameters have to be included. Therefore, fixed times and proportional times are required. The former is denoted by tf_i whilst the latter by tp_i.

It is also necessary to be able to specify the deadlines for the achievement of each target. This is denoted by Ut_j, and the time at which each resource will be available as soon as possible must also be known. These times are described by parameters Lt_j. A material m_j is only available at a time tm_j that is not smaller than its earliest availability (Lt_j), but not more than the defined deadline (Ut_j). That is:

$$\forall m_j \in M : Lt_j \leq tm_j \leq Ut_j \tag{1.6}$$

The start time to_i of activity $o_i \in O$ cannot precede the time tm_j of any precondition m_j, i.e:

$$o_i = (\alpha, \beta) \in O, \forall m_j \in \alpha_i : to_i \geq tm_j \tag{1.7}$$

The availability time tm_j of any m_j consequence of an activity o_i must not precede the completion of the duration $tf_i + x_i \cdot tp_i$ from the start time to_i of the activity, i.e:

$$o_i = (\alpha, \beta) \in O, \forall m_j \in \beta_i : tm_j \geq to_i + tf_i + x_i \cdot tp_i \tag{1.8}$$

1.5 Applications of the P-graph methodology, research trends

The P-graph methodology has recently been widely used to study and optimise production systems under different conditions. Friedler et al. have defined the application areas as follows [37]: (1) industrial applications: PNS, process integration, and improvement; (2) supply chains, logistics, and production scheduling; (3) sustainability assessment and circular economy; (4) reliability, resilience and risk assessments; (5) non-conventional applications; (6) extension of the model and software implementation; (7) novel directions. Some of the work published in recent years is mentioned below, illustrating each area of research. A more detailed overview can be found in [37], which provides an up-to-date summary of the different research directions.

One of the main advantages of the P-graph framework is that it provides the best or N-best solutions based on the structural properties of the problem to be optimised. Many scientific papers have been published in this area since the 1990s. The framework has also been applied in automated synthesis of process-networks by the integration of P-graph with process simulation, which enhances the accuracy of the physicochemical models [77]; in wastewater analysis and synthesis [102]; in a study how to start a reaction pathway with synthesis technique [65]; in connection with the synthesis of heat exchanger networks [75, 73, 72], and also in renewable energy storage and distribution scheduling [14].

Most applications can be found in supply chains, logistics and scheduling. Several works have been published addressing energy consumption in production systems

[66, 31]; dealing with scheduling field service operation and custom printed napkin manufacturing [42, 41].

Sustainability is another critical topic to be considered in process synthesis, and when considering the connectivity of energy, water and food [15, 47].

Another exciting line of research – and this is the focus of my dissertation – concentrates on how the reliability of complex systems and processes can be calculated algorithmically and integrated into optimization models [74, 60].

Several extensions of the traditional P-graph technique have appeared in recent years. Kalauz et al. have extended the mathematical model to handle time-dependent activities [54]. Nagy et al. [71] and Ercsey et al. [33] studied the bus transport process network synthesis problem and presented interesting real-world results in this area. Bartos and Bertok have investigated the possibilities of parallelising the developed solving algorithms and have supported their accuracy with case studies and analyses [9], while Heckl et al. have provided a modelling technique for operating units with flexible input ratios [32].

In addition to the research mentioned above, new extension opportunities have emerged, which foresee several future-oriented results.

Chapter 2

P-graph-based reliability optimisation: Redundancy allocation in energy systems

As most of the energy production and transformation processes are safety-critical, it is vital to develop tools that support the analysis and minimisation of their reliability-related risks. The resultant optimisation problem should reflect the structure of the process which requires the utilisation of flexible and problem-relevant models. This chapter highlights that P-graphs extended by logical condition units can be transformed into reliability block diagrams, and based on the cut and path sets of the graph a risk model can be extracted which opens up new opportunities for the definition optimisation problems related to reliability redundancy allocation. Risk models can be formalised by polynomials (polynomial risk model), where the exponents of the polynomial represent the number of redundant units, the cost function of the reliability redundancy allocation problem as a non-linear integer programming model can be formalised. The cost function handles the costs associated with consequences of equipment failure and repair times. The applicability of this approach is illustrated in a case study of the chapter related to the asset-intensive chemical, oil, gas and energy sector. The results show that the proposed algorithm is useful for risk-based priority resource allocation in a reforming reaction system. In the second part of the chapter a novel multi-objective optimisation based method is developed to evaluate the criticality of the units and subsystems. The applicability of the proposed method is demonstrated using a real-life case study related to a reforming reaction system. The results highlight that P-graphs can serve as an interface between process flow diagrams and polynomial risk models and the developed tool can improve the reliability of energy systems in retrofitting projects.

2.1 Review of redundancy allocation in energy systems

Retrofitting in the energy industry is important to improve the efficiency of power plants [93], increase energy production [5] and reduce emissions [104]. As safety-critical systems are retrofitted, optimisation demands a critical degree of attention [35]. Moreover, both in terms of design and retrofit of new technologies, a highlighted goal is to increase reliability and reduce maintenance costs, e.g. by increasing the maintenance cycle time. Therefore, operational excellence in the asset-intensive chemical, oil, gas as well as energy sectors should also be ensured by risk-based optimal design and maintenance planning. Redundancy allocation is widely used to identify critical elements where the reliability of the system at minimum cost can be maximised by redundancy. [76]. Many scientific articles highlight the importance of reliability-based studies. Such scientific results can also be found in graph-based environments [60].

In the present section, an overview of recent developments, trends and challenges in the synthesis, design and operation optimisation of energy systems with special attention paid to uncertainty, reliability, maintenance and social aspects. The most important modelling techniques and algorithms of recent years and decades are presented and typical structures described that are the starting points for designing and operating safety-critical systems. Moreover, motivated by deficiencies and current research trends, a novel multi-objective integer nonlinear optimisation method for minimising cost whilst maintaining a determined level of reliability is presented. The model of the reliability-based redundancy allocation problem is based on a polynomial risk model extracted from the path and cut sets of the flexible P-graph representation of process optimisation problems [38]. The main benefit of the P-graph based technique is that the polynomial risk model can be generated algorithmically based on the cut and path sets of the P-graph representation.

In the next section, first, the P-graph-based design of energy systems is described in Section 2.1.1. This is followed by a brief introduction to the literature review of redundancy allocation in energy systems in Section 2.1.2.

2.1.1 P-graph-based design of energy systems

Several articles have been published in recent years to design optimal energy systems based on P-graph methodology, e.g. synthesis of chemical and energy conversion systems, supply chains, waste and resource management, modelling chemical reactions, and discrete event simulation-based decision-making [94].

In a P-graph, the input and output elements, as well as the technological units of the energy system must be identified where the relationships between the subsystems and material flows provide a high-level representation of the processes. By visualising the P-graph representation of a process, the horizontal bars represent the operating units, while the solid circles indicate the material streams. A group of papers that focus on P-graphs support the system and supply chain design, redesign and optimisation problems [90, 58] and also very specific part of the field such as the asset management and retrofitting problem, in which an analysis of the investment planning concepts is provided [95].

In the case of energy systems, the process flow diagram of a power plant can be conveniently transformed into a P-graph, thus, the total site heat integration [99], carbon footprint targets and other technological aspects can be handled via mixed-integer linear programming (MILP) models. Related to the P-graph representation and methodology, the criticality analysis based system design is demonstrated by Benjamin et al. [12] using a risk-based matrix of the integrated bioenergy systems. The methodology applied combines the original approach of criticality analysis and advantage of the algorithmic characteristics of the P-graph framework and the methodology is represented by a bioenergy park and a palm oil-based integrated biorefinery case study.

Moreover, this methodology also allows for the handling of such cases where the demand for outputs and the availability of inputs are not constants and can change over time. This is a typical requirement of energy systems which can be managed by introducing new variables and constraints. Based on the defined superstructure, the mathematical model of the problem can be written and the solutions are generated automatically. The described P-graph-based methodology for the design and optimisation of power systems is well represented via the optimal planning of carbon capture and storage deployment in the power generation sector [19] and the case study of an energy system where the heating requirements of a farm using alternative inputs are taken into account [91].

In some special cases, the superstructure can be omitted and superstructure-free synthesis and optimisation conducted as the real-life case study of distributed energy supply systems presented [97], where in contrast to the traditional MILP solver, heuristics and evolutionary algorithms support the identification of the best solutions. In many cases, problems related to synthesis are subject to uncertainties, where the product demands and/or availability of raw materials are not exact values. The various extensions of the P-graph also provide a technique to handle such cases, e.g. fuzzy constraints and ranges [7] as well as random variables [90] are able to handle uncertainty events.

However, due to the safety-critical nature of energy systems, reliability is of crucial importance and a need for a systematic P-graph-based methodology explicitly for the reliability-based analysis and optimisation of such systems is present. In this chapter such a methodology is presented.

2.1.2 Literature review of redundancy allocation in energy systems

Before going into detail, an overview of redundancy allocation in energy systems is provided. For mapping the literature, I searched for publications related to the topic on Scopus with properly chosen keywords yielded 168 papers. Off-topic results were filtered whilst the remaining themes were separated considering the properties of how they connect to energy systems, how practical they are, and what are the benchmark topics. The results of text mining highlight the frequently co-occurring keyword pairs in the analysed abstracts of these selected publications as depicted in Figure 2.1. The nodes represent the keywords of each abstract and two nodes are connected by a link if they frequently co-occur among the keywords of abstracts. The size of the nodes is proportional to the frequency of occurrence of the related keyword, while the thickness of the edge is proportional to the frequency of co-occurrence of the keywords. The year of publication is indicated by the colour of the node.

Several types of redundancy strategies can be found in the literature. *Hot redundancy* is applied, when each component operates simultaneously, although only the primary is required. Nowadays this is almost the exclusive method in the safety-related processes. In the case of *warm redundancy*, the redundant element has a low load until the failure of the operating element. *Passive redundancy* is considered when the redundant element does not carry any load until the failure of the operating element. Finally, we talk about a *hot-standby* strategy, when the redundant element does not carry any load, but in case of a failure of the primary element, the operation can be switched to the redundant one to keep the system operational.

A general additional feature to the redundancy of structural schemes is the definition of a k-out-of-n system (hereinafter referred to as *koon* system) having n components and failing if and only if at least k component fails. This structure has the benefit of having the opportunity of tuning the parameters k and n, where

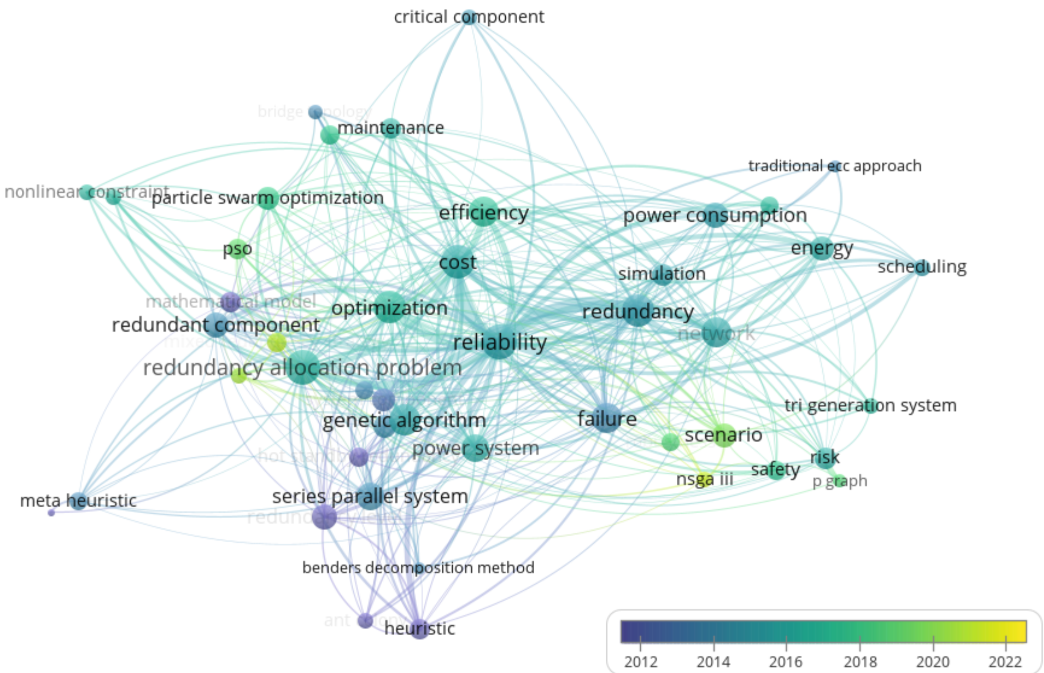

Figure 2.1: A network representing the topic of redundancy allocation in the literature. Each node represents a keyword of the abstracts and two nodes are connected by an edge if the words frequently co-occur in the abstracts. The colours of the nodes indicate the year of publication

k and n are the number of operating and the number of the overall components, respectively.

The provision of redundant critical process units/components can significantly reduce the operational risk of these systems. As such modifications of the technology require additional investment and maintenance costs, it is beneficial to formalise the reliability redundancy allocation problem as an optimisation task. The optimisation of a boiler-feed water treatment plant, where the maximisation of reliability with cost constraint was considered [55]. The mathematical model of the problem is given as a k-out-of-n system. The publication incorporated the identification of the components of the system including the weak links where the application of redundancy would be appropriate and useful. A P-graph based nonlinear integer programming model of the reliability-redundancy allocation problem was introduced in [88] where the reliability of the given system was maximized subject to some cost constraints. The redundant process units were represented by logical nodes in the P-graphs and Mesh Adaptive Direct Search (NOMAD) black-box algorithm was used to solve the developed mathematical model.

Due to the optimal redundancy allocation is an NP-hard problem, most research concerns the development of genetic algorithms and partical swarm optimisation-based solutions.

The non-linear integer programming problems are often formalised and solved by genetic algorithms (GAs). E.g. GAs were successfully applied for the optimisation of active and standby redundancy strategies [76] and redundancy allocation in a multi-state power system based on cost and availability requirements [34].

Redundancy allocation in energy systems has become a widely applied benchmark problem for the development of particle swarm optimisation (PSO) [68] and ant colony optimisation (ACO) solutions [79]. In these works, the total system reliability is maximised while total system cost and weight are constrained [70].

The original idea to take into consideration multiple objectives during the redundancy allocation problem dates back to the 1970s. A multi-objective formulation of a reliability allocation problem to maximize system reliability and minimize system cost was first formulated by Sakawa [84], while Inagaki *et al.* used an interactive optimisation approach to design a system with minimal costs and weight [53]. Moreover, multi-objective optimisation can also be applied to determine the optimal replacement age in the presence of competing criteria [52]. A multi-objective reliability allocation problem for a series system with time-dependent reliability was first presented in [25]. A novel multi-objective particle swarm optimisation algorithm and a multi-objective mathematical method were proposed by Dolatshahi-Zand and Khalili-Damghani for the optimisation of a SCADA water resource management control center [27]. A meta-heuristic particle swarm optimisation-based strategy is applied to optimise the redundancy allocation problem of multi-state systems with bridge topology in the case of a coal conveyor multi-state system with limited system availability and a limited budget [100].

The ant colony technique can also be applied to solve a multi-objective optimisation problem in the context of the redundancy and maintenance of a multi-state *koon* system [1]. Recently de Paula *et al.* proposed a solution for the redundancy allocation problem by applying a stochastic Markov Chain-based approach using Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [23].

The redundancy allocation problem frequently occurs in energy systems. Recently the problem was investigated in terms of generators and transformers in power systems [64]. With an ever-increasing emphasis on renewable energy systems, the reliability of wind farms has also become a cardinal issue [34, 1, 2]. The redundancy allocation of turbocharger overspeed protection has become a widely applied benchmark problem [101, 46, 30, 29] similar way to the integrated design of a steam turbine configuration for a biomass-based tri-generation system [4]. Moreover, the redundancy of brake lining [81], a coal conveyor belt system in a power system [100] and a pressurized water reactor cooling loop system [3] are also taken into consideration.

The previously presented overview highlighted that in a sophisticated model-based optimisation methodology, the costs associated with maintenance and the consequences of equipment failure should be structured according to the hierarchy of the assets, and the time-dependence of the failure probabilities as well as maintenance activities should also be considered. In the following, a method that meets these requirements will be presented.

2.2 Description of reliability-focused structural characteristics of complex systems using classical frameworks

Fault tree analysis is the most commonly used method in risk and reliability calculation. This technique can be successfully applied in many fields, whether engineering or IT systems, but it is also an essential methodology for performing risk analysis tasks [63, 82].

The analysis starts from a hypothetical system failure, a TOP event, and progressively identifies the component and subsystem failure modes that lead to the occurrence of that event.

The methodology is supported by a tree structure-like graphical representation (fault tree), which can complement reliability calculations.

During the analysis, the methodology aims to identify all failures and combinations of failures leading to the TOP event and their causes; to detect particularly critical events and event chains; to calculate reliability figures along the branches of the fault tree; to identify failure mechanisms.

In principle, fault tree analysis involves four main steps: Formulate the problem and select the TOP event; Prepare a fault tree describing the problem; Analyse the fault tree; finally, evaluate the results.

Fault trees are constructed with various event and gate logic symbols. Although many events and gate symbols exist, most fault trees can be built using TOP or Intermediate event, inclusive OR gate, AND gate, and basic event.

Figure 2.2 illustrates the fault tree itself and its elements.

A fault-tree-based representation can be used to quantify the risk analysis. The risk of a TOP event occurring can be characterised using probability tools. This step requires identifying the so-called cut sets of the fault tree. A cut set is any group of events that will cause the TOP event to occur if they all happen. The minimum cut set is the set of cut sets with the smallest number of elements.

Assume that the minimal cut set is unique. In this case, if the probabilities of occurrence of the events in the minimal cut set are $p_1, p_2, \ldots, p_n$, then the probability of occurrence of the TOP event is the product of these probabilities, i.e. $p_1 * p_2 * \ldots * p_n$. It implies that the probability that the TOP event does not occur can be easily computed, and the procedure can be generalized to cases with several minimal cut sets.

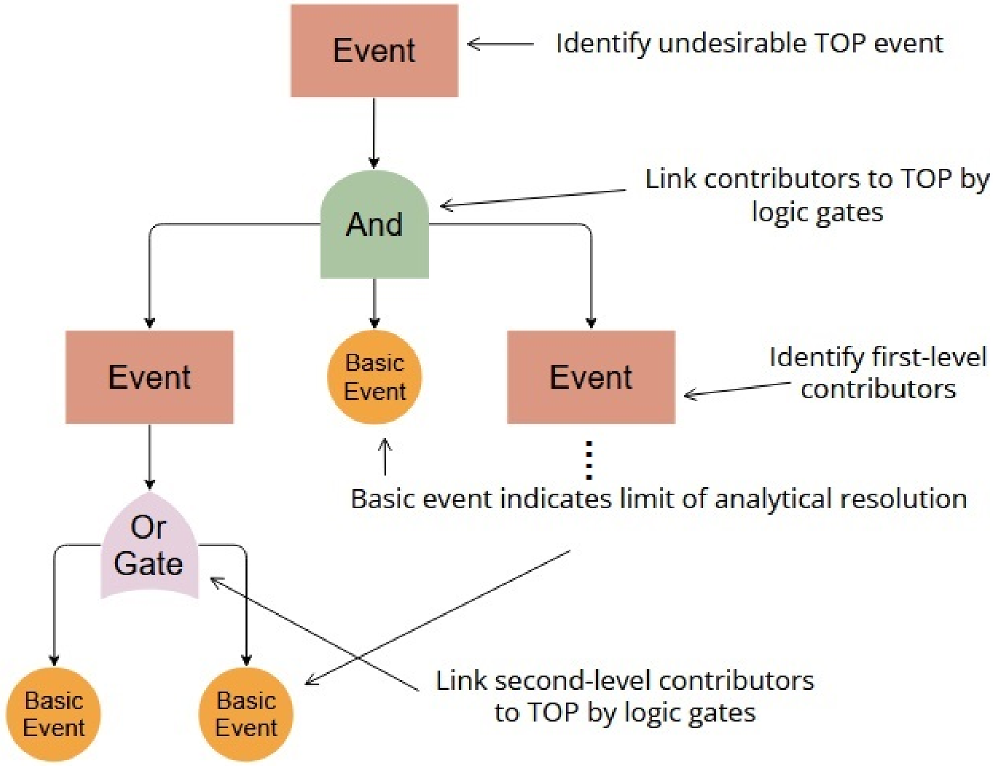

Figure 2.2: Elements of a fault tree

A model describing the desired system state or event can be written on the analogy of a fault tree. The success tree can be generated from the fault tree by simple transformation steps. The way to do this is to replace all OR gates with AND gates, all AND gates with OR gates, and the TOP event reflects the desired success state.

In addition, other system description methodologies exist; the reliability block diagram (RBD) Fig. 2.3 represents the contribution of subsystems of complex systems to the overall system. It describes the connections between the elements; thus, the method can be used to predict the system's availability and analyse the criticality of the subsystems.

In Chapters 2 and 3, the methodology presented here will be adapted to those cases where P-graph-based models describe the structural characteristics of a system. For example, details of fault-tree and success-tree-based risk analysis can be found, among others, in [43, 82].

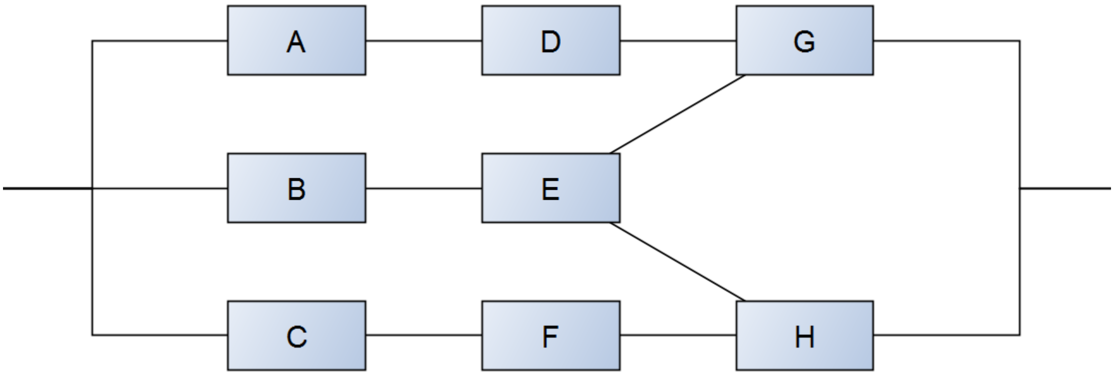

Figure 2.3: Example of a Reliability Block Diagram

2.3 Notations

To summarize, the list of functions, variables, and parameters are as follows:

Functions:

φ	–	system structure function
P^{UB}	–	upper bound of reliability of the system
P^{LB}	–	lower bound of reliability of the system

Variables:

e	–	vector, representing the functioning-or-failed condition of components
e_i	–	represents condition of ith component
d_i	–	number of redundant i units
o^*	–	set of materials and operating units in the optimal solution
z^*	–	optimal value of the objective function

Parameters:

c	–	number of the components
C_fm	–	fixed cost of maintenance
CV	–	variable cost of maintenance per day

DT	–	downtime
M	–	set of materials
O	–	set of operating units
P	–	set of products
R	–	set of raw materials
(P, R, O)	–	PNS problem
$Limit_{risk}^{Upper}$	–	Upper bound of the acceptable risk
$Limit_{component}^{Upper}$	–	number of spare components
M	–	materials
MC	–	maintenance cost
π_i	–	minimal path i
PL	–	production loss
$PLPD$	–	production loss per day
n_p	–	number of minimal paths
ϑ_i	–	cut set i
n_c	–	number of cut sets

2.4 P-graph based representation of energy systems

The focus of this chapter is the safety-critical optimal design of complex process systems. For this purpose, the reliability-redundancy allocation task is interpreted as a process network synthesis (PNS) problem and a widely applicable method is proposed for the evaluation of the reliability of systems represented by P-graphs.

Most business, manufacturing and technological processes can be depicted by P-graphs. Although this representation was primarily used to describe production processes in the early 1990s [38], nowadays, several applications are known that consider energy technology networks and many other problems, e.g. the design process of wastewater treatment systems [13] and development of production processes [10].

In early P-graph studies only material transformation steps were symbolised by an operating unit, recently the whole concept has been extended to include the modelling and analysis of workflows. Accordingly, the logical connections between the elements (logical 'AND', logical 'OR') can be represented by the operating units and material-type nodes (see Figure 2.4). The transformation between success trees, reliability block diagrams and P-graphs can easily be given since the operating units of a P-graph represent the functionalities of the components, and materials are used to introduce elementary faults into the model, as represented in Figure 2.5.

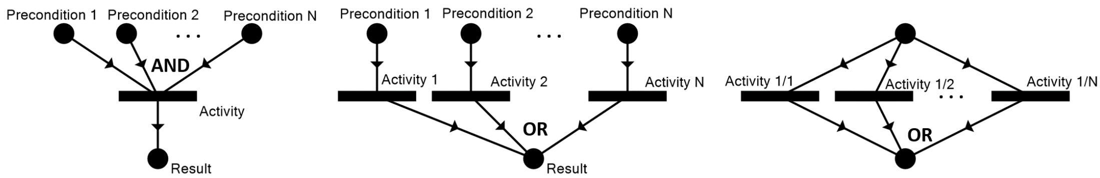

Figure 2.4: Representation of (a) AND and (b) OR dependencies as well as (c) the redundancy of activities as OR connections

The path and cut sets identified in a P-graph provide the opportunity to perform reliability-based analyses and extend previous cost/profit optimisation procedures. Note, that in complex systems it is expedient to construct a P-graph by defining subsystems because a clear representation can be given. Furthermore, any part of the P-graph can be examined separately, thus, further reliability analyses can be executed to improve redundancy and reliability even more.

The algorithms that support the structural analysis of P-graphs are extended in the next subsection and a reliability-based technique will be constructed using cut and path sets of P-graphs.

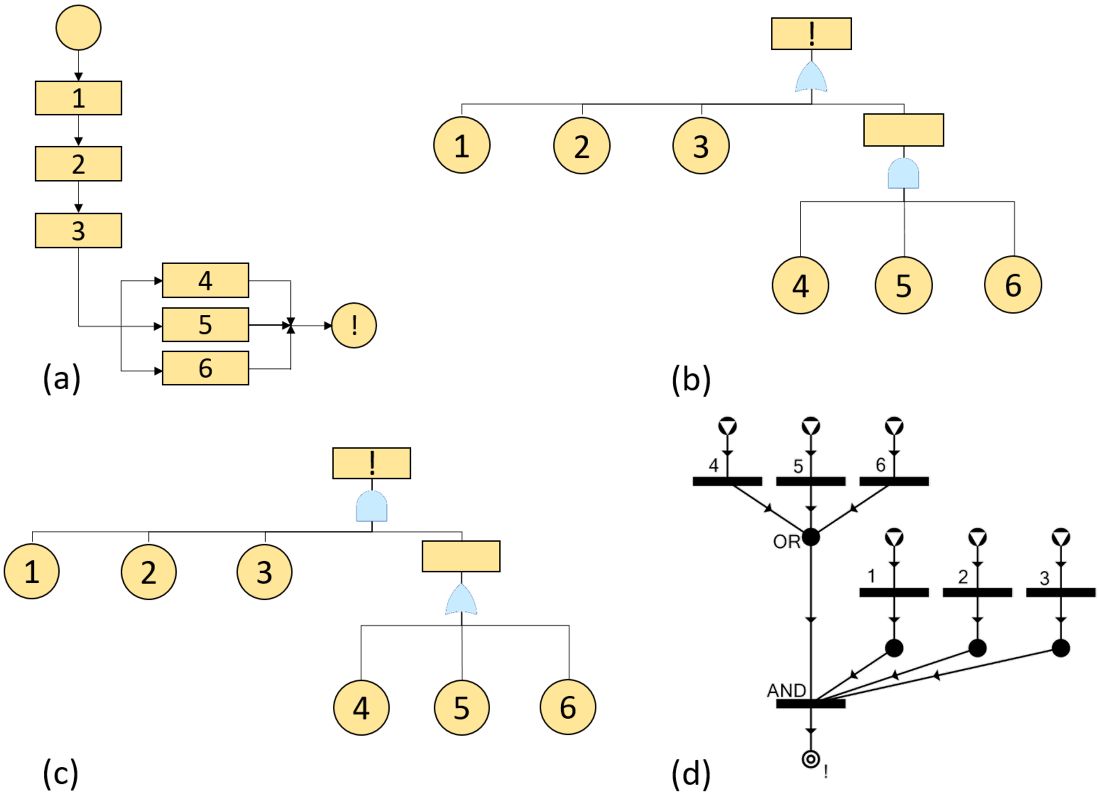

Figure 2.5: Example of a (a) Reliability block diagram, (b) Fault tree, (c) Success tree, and (d) P-graph representation. As can be seen, P-graphs can represent both reliability block diagrams and success trees

2.5 Reliability analysis for time-independent case based on the cut and path sets of P-graphs

The focus is the safety critical optimal design of complex process systems. For this purpose, the reliability-redundancy allocation task is interpreted as a process network synthesis problem and a widely applicable method is proposed for the evaluation of the reliability of systems represented by P-graphs.

2.5.1 Mathematical background

It is assumed that the system is built from c components. Due to failures, some of these components do not perform their required functions within specified performance requirements, which can result in the whole system losing its functionality.

The functioning-or-failed condition of components is represented as an

$$e = [e_1, \ldots, e_i, \ldots, e_c]^T$$

vector, where $e_i = 1$ represents that the i -th unit is functioning, while $e_i = 0$ represents the failure of the i-th component. The system structure function is a Boolean function that maps $\{0,\, 1\}^c$ into $\{0, 1\}$, which represents $e_0 = \varphi\left(e\right)$, assuming the whole system is functioning correctly. When the components of the system are in series then

$$\varphi\left(e\right) = e_0 = e_1 \cdot \ldots \cdot e_c,$$

but when in parallel

$$\varphi\left(e\right) = e_0 = 1 - \left(1 - e_1\right) \cdot ... \cdot \left(1 - e_c\right).$$

The reliability of the system is equivalent to the probability of the system properly functioning, $P\left(\varphi\left(e\right) = 1\right)$. The structure function is usually represented as reliability block diagrams.

The reliability block diagram of the system is a labeled random graph, where the nodes e_i represent the nodes of random variables indicating the i-th node is present in the graph. A path in a graph is a sequence of alternating adjacent nodes and the links joining them, beginning and ending with a node. Therefore, when a path to the end of the reliability block diagram exists through the sets of operating nodes/units, then the system is working properly. A path is referred to as minimal if it contains no proper subset that is also a path connecting the same two nodes. As a result, the set of minimal paths defines the set of operating units that ensure the operation of the whole system. Since there can be several minimal paths, $\pi_1, \ldots, \pi_{n_p}$, the system functions when at least one path is available, so the (upper bound of) reliability of the system is:

$$P^{UB}\left(\varphi\left(e\right)\right) = 1 - \prod_{k=1}^{n_p}\left[1 - \prod_{i\in\pi_k} P\left(e_i = 1\right)\right] \tag{2.1}$$

A cut is a set of nodes and links whose removal from the graph disconnects the beginning and ending nodes, so the sets of minimal cuts connect the sets of units whose failure results in the failure of the whole system. Namely, the system fails if at least one of the minimal cuts consists entirely of non-functioning units. Since several cut sets can exist, $\vartheta_1, \ldots, \vartheta_{n_c}$, therefore, the lower bound of the reliability of the system is:

$$P^{LB}\left(\varphi\left(e\right)\right) = \prod_{k=1}^{n_c}\left[1 - \prod_{i\in\vartheta_k}\left[1 - P\left(e_i = 1\right)\right]\right] \tag{2.2}$$

A path in the P-graph defined between a raw material flow and a product is a sequence of alternating adjacent material and operating unit nodes and the links joining them, beginning and ending with a material node. The analogy between a P-graph and a success tree can easily be realised since the operating units of a P-graph can represent the functionalities of the components, and the materials denote the faults in the model. In order to illustrate the analogy the following simple example should be considered. Since an operating unit represents a device to which an activity is associated, a heat exchanger corresponds to an operating unit in a P-graph model. Representing the temperature exchange of the air flowing through the heat exchanger from cold to hot or vice versa this unit requires input material to be transformed into another output material corresponding to the cold and hot air.

All the feasible solution structures can be generated automatically based on the initial P-graph according to the SSG algorithm [40], which also defines paths from the raw materials to the products. The elements of a feasible solution structure ensure the uninterrupted operating status of the system, and the set of the operating units provides one element of the path set at the same time. All elements of the path set can be obtained by producing all the feasible solution structures. In order to determine the reliability of a system, the minimal path sets are required, so a novel algorithm is needed to generate the elements of this set based on the initial structure. A minimal path set is a minimal set of components whose simultaneous work ensures that the system works properly. The set of minimal path sets which are required for the analysis of the reliability of the system can be given by the Path Set Generator Algorithm (2.1). The input of Algorithm 2.1 is a graph defined by (m, o), where m denotes the set of materials and o represents the set of operating units. The algorithm produces a minimal path set by examining sub-problems starting with the products represented by P. The bottom-up construction of the algorithm results in possible feasible solution structures that are also part of the minimal path set. Since an operating unit is defined by its input (α) and output (β) material sets, the minimal path set is also given by a set of (m, o) pairs.

Such a P-graph is shown in Figure 2.6 where the set of materials is $M = \{A, \dots, F\}$, the raw materials are $R = \{A, B, C, E\}$, and the set of products is represented by a single element in $P = \{F\}$. The operating units are as follows: $O = \{O_1, O_2, O_3, O_4\}$, where $O_1 = (\{A\}, \{D\})$, $O_2 = (\{B\}, \{D\})$, $O_3 = (\{C, D\}, \{F\})$ and $O_4 = (\{B, E\}, \{F\})$. There are 7 different feasible solution structures which can easily be seen: $Str_1 = \{O_1, O_3\}$, $Str_2 = \{O_2, O_3\}$, $Str_3 = \{O_1, O_2, O_3\}$, $Str_4 = \{O_4\}$, $Str_5 = \{O_1, O_3, O_4\}$, $Str_6 = \{O_2, O_3, O_4\}$ and $Str_7 = \{O_1, O_2, O_3, O_4\}$; only three of which, namely Str_1, Str_2, and Str_4, are the elements of the minimal path set.

Algorithm 2.1: Path Set Generator

Input : (m, o): P-graph
Output : minimal path sets

begin
min-path-sets := $\varnothing$
subproblems := $(P, \varnothing, \varnothing, (O \setminus o))$
while *subproblems* $\neq \varnothing$ **do**
  let $(p, p^+, o^+, o^-) \in$ subproblems , where $|(o^+)|$ is minimal
  subproblems := subproblems $\setminus (p, p^+, o^+, o^-)$
  if $\{(m, o) \in$ *min-path-sets* $|o \subseteq o^+\} = \varnothing$ **then**
    if $p = \varnothing$ **then**
      $\psi := \cup_{(\alpha,\beta)\in o^+}(\alpha \cup \beta)$
      min-path-sets := min-path-sets $\cup\{(\psi, o^+)\}$
    else
      let $x \in \{\hat{x}|\hat{x} \in p$ and $|(\alpha, \beta) \in o : \beta \cap \hat{x} \neq \varnothing|$ is minimal$\}$
      $o_x := \{(\alpha, \beta) \in o : \beta \cap x \neq \varnothing\} \setminus o^-$
      $o_{xb} := o_x \cap o^+$
      $C := \wp(o_x \setminus o_{xb})$
      if $o_{xb} = \varnothing$ **then**
        $C := C \setminus \{\varnothing\}$
      end
      for *all* $c \in C$ **do**
        $\hat{p} := ((\cup_{(\alpha,\beta)\in c}\alpha) \setminus p^+ \setminus \{x\} \setminus R$
        $\hat{p}^+ := p^+ \cup \{x\}$
        $\hat{o}^+ := o^+ \cup c$
        $\hat{o}^- := o^- \cup (o_x \setminus o_{xb} \setminus c)$
        subproblems := subproblems $\cup\{(p, p^+, o^+, o^-)\}$
      end
    end
  end
end
return min-path-sets
end

Algorithm 2.2: Structural Reliability Calculator

Input : (m,o): P-graph, p_o: set of reliability of operating units, where p_{o_i} gives the reliability of the operating unit i
Output : reliability of the whole system

begin mint-path-sets:=Path Set Generator $((m, o))$
return $1 - (\prod_{k=1}^{min-path-sets}(1 - \prod_{i \in \prod_k} p_{o_i}))$ end

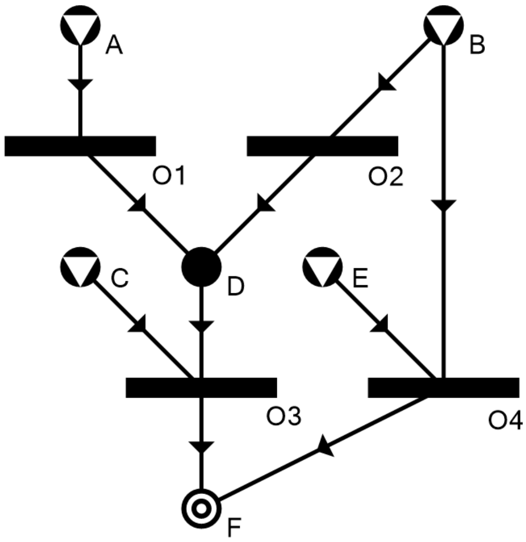

Figure 2.6: Illustrative example of a P-graph representing the minimal path and cut sets.

By applying the elements of the minimal path set, the reliability of the system can be calculated by Algorithm 2.2. Although the complexity of the Algorithm 2.1 is exponential, it is essential to emphasize that practical experience shows that the procedures developed for P-graph-based solutions are complete in polynomial time. Friedler and colleagues gave an example of this in their earlier work [38].

Algorithm 2.1 is able to determine path sets even if several TOP events exist, in contrast to the traditional fault tree and success tree techniques where exactly one TOP event is determined. In the case of multiple TOP events, in the representation of the P-graph an operating unit can symbolize the 'AND' relationship between these events.

A close relationship exists between minimal path and minimal cut sets: the former defines the operational probability of the overall system, while the latter indicates its complementarity, i.e. the risk of failure. The system fails if at least one of the minimal cuts consists entirely of non-functioning units. The minimal cut sets are created in such a way that all logic gates are exchanged, i.e. each AND becomes OR and vice versa, thus, the P-graph can be constructed accordingly. Note that the minimal path set of the example shown in part *d* of Figure 2.5 is

$$\{\{1,2,3,4\},\{1,2,3,5\},\{1,2,3,6\}\},$$

while the minimal cut set is

$$\{\{1\},\{2\},\{3\},\{4,5,6\}\}.$$

The reliability of the entire system can be characterised by a polynomial expression, as the reliabilities are multiplied when the elements are connected by AND connections, while logical OR connections aggregate the different sets. As an increase in the reliability of the system by introducing redundant elements is desired, the above equations can be written as follows:

$$P^{UB}\left(\varphi\left(e\right)\right) = 1 - \prod_{k=1}^{n_p}\left[1 - \prod_{i\in\pi_k} 1 - \left[1 - P\left(e_i = 1\right)\right]^{d_i}\right] \tag{2.3}$$

$$P^{LB}\left(\varphi\left(e\right)\right) = \prod_{k=1}^{n_c}\left[1 - \prod_{i\in\vartheta_k} \left[1 - P\left(e_i = 1\right)\right]^{d_i}\right] \tag{2.4}$$

where d_i represents the number of units. The evaluation of the risk associated with the failure of the system requires the calculation of the economic consequence of equipment failures. In our study, the cost of the required maintenance cost (MC) and the cost of the production loss (PL) were calculated:

$$MC = C_f m + DT \cdot CV \tag{2.5}$$

$$PL = DT \cdot PLP \tag{2.6}$$

where $c_f m$ stands for the fixed cost of maintenance ($), DT denotes the downtime (number of days), CV represents the variable cost of maintenance per day ($ day^{-1}), and $PLPD$ is the production loss per day ($ day^{-1}). The risk of each subsystem is the product of its failure probability and consequences of failure. Based on this loss function and the polynomial reliability of the model, the following risk function can be determined, where o^* represents the set of materials and operating units involved in the optimal solution:

$$\begin{aligned} \sum_{(\alpha,\beta)=o_i \in o^*} & \left(cfm_i + DT_i \cdot CV_i\right)\left(1 - P\left(e_i = 1\right)\right) + \\ & + \left(DT_i \cdot PLPD_i\right)\left(1 - P\left(e_i = 1\right)\right)^{d_i} \quad \leq Limit_{risk}^{Upper} \end{aligned} \tag{2.7}$$

whose risk is inversely proportional to the reliability of the system:

$$z^* = P^{UB}\left(\varphi\left(e\right)\right) = 1 - \prod_{k=1}^{n_p} \left[1 - \prod_{i \in \pi_k} 1 - \left[1 - P\left(e_i = 1\right)\right]^{d_i}\right] \tag{2.8}$$

The risk always decreases by increasing the redundancy. However, the installation of additional components requires investment costs, resources for which are limited. As detailed information concerning the investment costs of the components is unavailable, the number of spare components is constrained:

$$\sum_{i=1}^{n} d_i \leq Limit_{component}^{Upper} \tag{2.9}$$

Based on these variables, a nonlinear integer programming model was defined, where the z^* objective function is maximised under the constraints related to the upper bound of the acceptable risk, $Limit_{risk}^{Upper}$, and the number of spare components (available investment costs) $Limit_{component}^{Upper}$.

2.5.2 Case Study

The applicability of the proposed methodology is demonstrated using data from a real-life case study related to the reforming reaction system in Sinopecs Luoyang Petrochemical Plant ([51]), which describes a real chemical system, where the authors provide the relationships between the system and its subsystems, as well as the reliability values of the subsystems.

The reliability and cost parameters of the subsystems of the process are given in Table 2.2. Instead of solving a process synthesis problem, in this study the P-graph of the process is obtained based on the success tree of the system (see Fig. 2.7). Since the data is aggregated to the subsystems, the reliability-redundancy allocation problem is also defined at this level (see Figure 2.8).

Based on the P-graph, the path sets were determined by the proposed minimal path set generation algorithm. Because of the specific topology of the graph, the minimal path set contains all the activities in the graph, therefore, $P^{UB}\left(\varphi\left(e\right)\right) = \prod_{i\in\pi_k} P\left(e_i = 1\right) = 0.009$. The Nonlinear optimisation by Mesh Adaptive Direct Search (NOMAD) black-box algorithm is used to solve this developed mathematical model. The algorithm defines a mesh with the discretisation of the space of variables and performs an adaptive search while the refinement of the mesh is also controlled ([6]). The solutions are verified by BARON ([83]) which is a computational system for solving nonconvex optimisation problems to global optimality. The reliability of optimal solutions for different constraints is presented in Table 2.3. The results show that by increasing the available budget, the reliability of the system is also increased, however, the number of redundant elements comprehensively determines the total cost and reliability. The results illustrate that the proposed methodology is applicable with regard to the risk-based resource allocation in the design of process systems.

Table 2.2: Reliability and cost parameters of subsystems (n=9)

#	Subsystem	Reliability ($P(e_i = 1)$)	cfm_i ($)	DT_i (day)	CV_i ($)	$PLPD_i$ ($)
1	1^{st} compressor subsystem	0.4208	2,173.9	1.5	144.93	43,478
2	Heating-reaction subsystem	0.4011	7,246.4	5.0	289.86	43,478
3	Heat exchanger subsystem	0.6088	2,898.6	3.0	289.86	43,478
4	Cooler subsystem	0.6801	1,449.3	2.0	289.86	43,478
5	Separation subsystem	0.9907	2,898.6	4.0	289.86	21,739
6	Pump subsystem	0.5722	724.6	1.0	72.464	0
7	2^{nd} compressor subsystem	0.7874	1,449.3	1.0	144.93	0
8	Absorber subsystem	0.6984	1,449.3	4.0	144.93	14,493
9	Instrument subsystem	0.4141	724.6	1.0	72.464	0

Table 2.3: Results of optimisation

#	$Limit_{risk}^{Upper}$	$Limit_{component}^{Upper}$	$d = (d_1, d_2, ..., d_9)$	Reliability of the system
1	110,000	15	(2,4,2,2,1,1,1,1,1)	0.0568
2	150,000	15	(2,3,2,1,1,2,1,1,2)	0,0879
3	180,000	15	(2,2,2,2,1,2,1,1,2)	0,0947
4	35,000	25	(5,6,4,3,1,1,1,3,1)	0,1514
5	50,000	25	(3,6,3,3,1,2,2,2,3)	0,3922
6	70,000	25	(4,4,3,2,1,3,2,2,4)	0,4563

2.6 Major results and related publication

In this section, a novel approach for safety-critical optimisation of process systems was presented. To represent redundant process units and to calculate the reliability of the system logical nodes to P-graphs were added. It was demonstrated that P-graphs extended by these logical condition units can be transformed into reliability block diagrams, and based on the cut sets and path sets of the graph

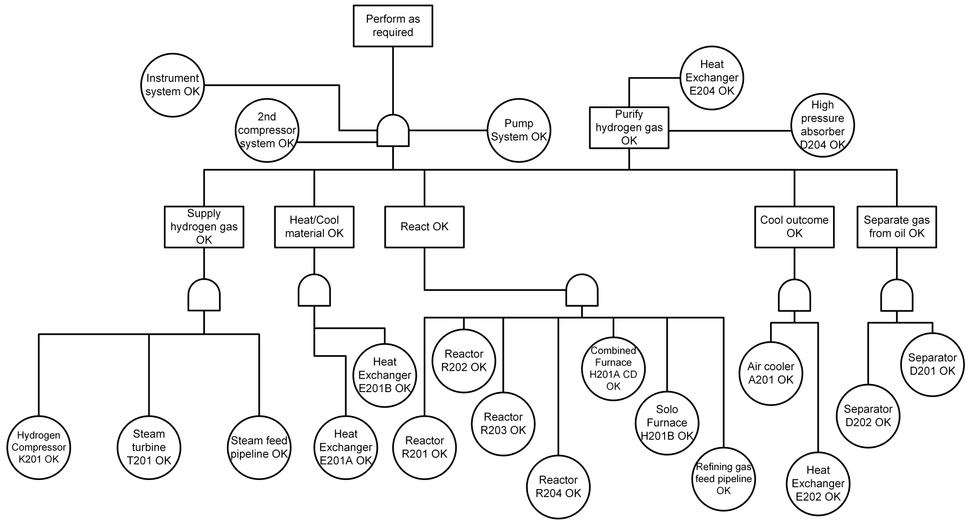

Figure 2.7: Success tree of reaction system published in [51]

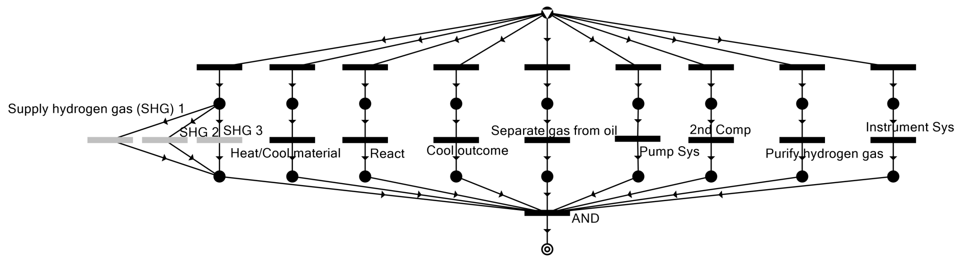

Figure 2.8: P-graph representing the subsystems of the reaction system. The figure also illustrates how redundancy is handled in the proposed framework

a polynomial risk model can be extracted. The cost function in terms of the reliability redundancy allocation problem was formalised as nonlinear integer programming model, where the integers are the exponents of the polynomial model that represent the number of redundant units. With the help of the NOMAD algorithm, the reliability under the constraints related to the investment costs and the acceptable risks associated with the consequences of equipment failure and repair times was maximised. The applicability of this approach was illustrated by a case study related to a reforming reaction system. In the next section, how the time-dependent reliability of the units will be incorporated into the model and how the proposed toolset can be used for the prioritisation of the maintenance work will be focused on.

Related publication

Süle Z., Baumgartner J., Abonyi J., 2018, Reliability - redundancy allocation in process graphs , Chemical Engineering Transactions, 70, 991-996 DOI:10.3303/CET1870166, Rank: **Q3**

2.7 Thesis 1

I have adapted the fault- and success tree-based methodology of reliability calculation to the P-graph framework. The developed approach allows the algorithmic reliability-based analysis of processes given by P-graph descriptions.

- I have developed an algorithm for generating minimal path sets of P-graph processes, which allows the calculation of process reliability.
- I have built a P-graph-based optimisation model to solve the reliability–redundancy allocation problem. The evaluation of the objective function in the implemented model is calculated by computing the minimal path sets of P-graphs.
- I have validated the results of the P-graph-based mathematical model by solving a real case study of the literature. Based on the success tree of a real reaction system, a polynomial risk model has been developed, and reliability optimisation, as well as, computation of the number of the redundant elements has been performed.

Chapter 3

P-graph-based risk analysis of k-out-of-n configurations

The design, operation and maintenance processes of safety-critical energy systems require careful planning, modelling and optimisation steps [57]. As is depicted in Figure 3.1, the proposed P-graph based method also follows a similar structure. In the following, the details of these steps will be presented. Because of unified terminology, we note that a system is a set of components that together perform specific objectives. The overall system consists of components, one element of which is referred to as a subsystem. These two terms are used in this sense hereinafter.

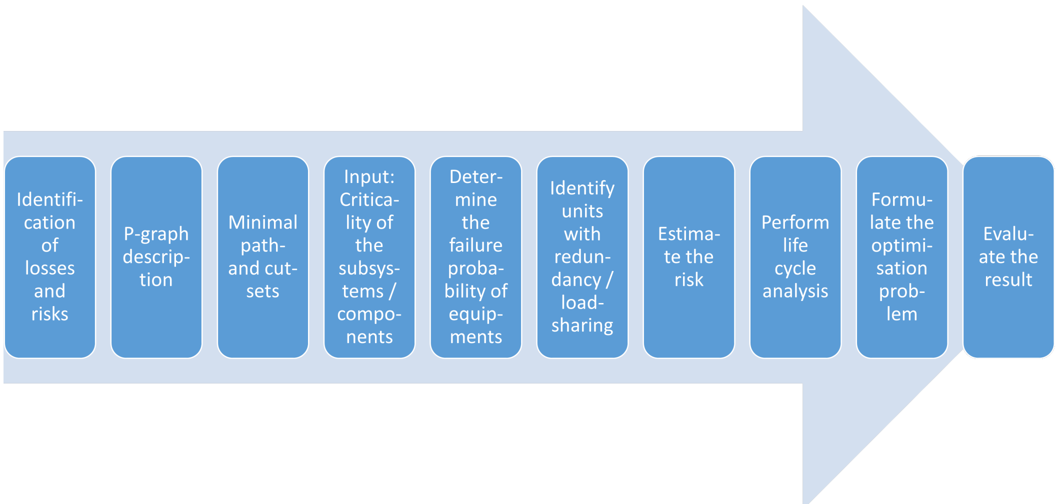

Figure 3.1: Steps of the proposed methodology of P-graph based risk analysis and redundancy allocation

3.1 Methodology

1. Identification of losses and risks: Identifying the relevant risk(s) of the overall system, where the value of a risk is $(1 - p) \times L$, where p represents the reliability of the system related to the TOP event (in other words p is the probability that the system is operational), and L shows the loss when the system fails.

2. Build the P-graph description of the system which presents the subsystems and relationships. In terms of integrated optimisation, the process has already been represented by a P-graph, but in retrofit cases, the P-graph structure has to be given.

3. Based on the structure of the system, efficient algorithms generate the minimal path and cut sets. As a result, the bottlenecks in the system become visible, as well as the role and dominance of the subsystems. The complexity of the algorithm is theoretically exponential but because it depends strongly on the system structure, practically results can be achieved in reasonable time.

4. Determine the criticality of the subsystems and components or the criticality of the initiating events, i.e. the value of criticalities constrain the optimisation within limits.

5. Determine the maximum acceptable failure probability of the system: $\frac{Risk_{Max}}{L}$, where $Risk_{Max}$ is the maximum acceptable risk.

6. Identify the critical units where redundancy can or should be applied: redundancy or load sharing can be applied at certain points of the system, which have to be given as inputs.

7. Estimate the risk that may result from the failures: the acceptable risks are given as constraints in the mathematical model.

8. Perform a life cycle analysis: the time-dependent fault probabilities can be calculated for the overall system and each subsystem.

9. Formulate a multi-objective optimisation problem: in general, a mixed-integer non-linear mathematical model is created and solved by heuristic algorithms.

10. Evaluate the results: The genetic/heuristic algorithm gives the optimal or a near optimal solution to the redundancy allocation and load-sharing problems.

A fault tree analysis-based methodology can also be useful for determining the reliabilities of the subsystems [56], [89]. A P-graph based model allows for many TOP events in the representation and reliability calculation, generation of the mathematical model and model solution can be performed algorithmically.

3.2 Notations

To summarize, the list of functions, variables, and parameters are as follows:

$\lvert . \rvert$	–	cardinality of a set
α	–	input set of materials of an operating unit
β	–	output set of materials of an operating unit
$\mathbf{e}(t)$	–	vector represents the functioning-or-failed condition of components
$e_i(t)$	–	vector represents the functioning-or-failed condition of component i
$e_0(t)$	–	vector represents the functioning-or-failed condition of the whole system
$f_i(\mathbf{x})$	–	objective function i
ic_i	–	investment and maintenance costs of the i-th component
$I_e(t)$	–	importance function of an elementary event e at time t
$I_k(t)$	–	importance function of the cut set ϑ_k at time t
k	–	number of operating elements in a *koon* system
L_i	–	loss when subsystem i fails
M	–	set of materials in the PNS problem

(m, o)	–	P-graph given by sets $m \in M$ and $o \in O$
N	–	number of types of components
n	–	number of overall elements in a *koon* system
n_c	–	the total number of minimal cuts
n_p	–	the total number of minimal paths
N_{gen}	–	number of generations in the genetic algorithm
N_{pop}	–	population of the genetic algorithm
O	–	set of operating units in the PNS problem
P	–	set of products in the PNS problem
$\phi(.)$	–	Boolean system structure function
π_i	–	minimal path set i
R	–	set of raw materials in the PNS problem
$R_i(t)$	–	reliability function of component i
$R_i^*(t)$	–	reliability function of the individual component i in a *koon* system
$R_0(t)$	–	reliability function of the whole system
$\wp(M)$	–	the power set of set M
$\mathcal{S}$	–	set of feasible solutions
t	–	time
T	–	lifespan
ϑ_i	–	minimal cut set i
$\times$	–	Cartesian product
$\mathbf{x}$	–	decision vector containing the decision variables

3.3 Reliability analysis for time-dependent case based on the cut and path sets of P-graphs

One of the objectives of this chapter is to generalise the mathematical model and approach presented in the previous chapter. This chapter of the thesis focuses on the time-dependent aspects of reliability calculation, which will provide us with more accurate analyses for the risk assessment of complex systems. The analysis will continue to be performed for systems written in P-graphs since this guarantees the algorithmic generation of the cut sets that form the basis of the reliability calculation in our models.

It is assumed that the system is built from N different types of components that deliver different degrees of reliability. Due to the complex interdependence of the elements when some of these components do not perform their desired functions within specified performance requirements, the whole system loses its functionality. Based on the time-independent methodology presented in Chapter 2, in the following I introduce the model of the time-dependent reliability. The functioning-or-failed condition of components is represented as an

$$\mathbf{e}(t) = [e_1(t), \ldots, e_i(t), \ldots, e_N(t)]^T$$

vector, where $e_i(t) = 1$ represents that the i-th unit is functioning at instance of time t, while $e_i(t) = 0$ represents that the i-th component at the instance of time t is not functioning properly.

The reliability

$$P(e_i(t) = 1)$$

of the components is represented by the reliability functions $R_i(t)$, $\forall i = 1, \ldots, N$.

In the case of complex systems the probability of the whole system $R_0(t) = P(e_0(t) = 1)$ functioning properly should be calculated based on the internal functional dependency of the components which is usually represented by the $e_0(t) = \phi(\mathbf{e}(t))$ Boolean function concerning the structure of the system that is an N dimensional binary vector ($\{0,1\}^N$) in order that the functionality of the whole system be $e_0(t) \in \{0,1\}$, so $P(e_0(t) = 1) = P(\phi(\mathbf{e}(t)) = 1)$.

Since several minimal paths, $\pi_1, \ldots, \pi_{n_p}$ can exist, the system functions when at least one path is available, so (the upper bound of) the reliability of the system is:

$$R_0(t) = P\left(\phi(\mathbf{e}(t))\right) = 1 - \Pi_{k=1}^{n_p}\left[1 - \Pi_{i\in\pi_k} R_i(t)\right] \tag{3.1}$$

as the $i \in \pi_k$ components of the path π_k are logically connected in series and the paths are logically connected in parallel to each other. This polynomial expression of the reliability of the entire system reflects that reliabilities are multiplied when the elements are connected by AND connections, while logical OR connections aggregate the different sets.

3.3.1 Identification of the critical elements in P-graphs

Since several minimal cut sets can exist, namely $\vartheta_1, \vartheta_2, \ldots, \vartheta_{n_c}$, the probability of the occurrence of a given cut set may be an important issue [43]. If all the events in the cut set ϑ_k occur, the TOP event is also produced, so the occurrence of the TOP event is mapped by an AND gate, where the events from the cut set ϑ_k are the inputs. Accordingly, the probability that the cut set ϑ_k occurs at time t is:

$$P(\vartheta_k \text{ occurs at time } t) = \prod_{i\in\vartheta_k} P(e_i(t) = 1) \tag{3.2}$$

A minimal cut set defines a number of important features of the whole system. Namely,

- a cut set with many elements indicates low risk for the overall system;
- a cut set with few elements indicates high risk for the overall system;
- many cut sets indicates high risk for the overall system;
- a few cut sets indicates low risk for the overall system;
- a cut set with only one element indicates a single point of failure which means that activation of a single elementary fault may result in system failure.

Let I_k be the importance of the cut set ϑ_k at time t. Assuming the cut set ϑ_k occurs at time t, the probability that ϑ_k induces the TOP event at time t is:

$$I_k(t) = \frac{P(\vartheta_k \text{ occurs at time } t)}{P(\text{TOP event occurs at time } t)}. \tag{3.3}$$

The importance of the elementary events of the cut set needs to be measured, since it can provide some useful information when the aim is to design a redundant system for maximising its reliability with a given budget. The probability that event e contributes to the TOP event at time t if it occurs is:

$$I_e(t) = \sum_{k \in \{i | e \in \vartheta_i, i=1,\ldots,n_c\}} I_k(t). \tag{3.4}$$

Note that when calculating the middle and TOP event reliabilities, the independence of vulnerabilities of the elementary events is assumed. Otherwise, the calculated reliability may be significantly in correct.

3.3.2 Multi-objective formulation of the *koon* redundancy allocation problem

In critical cases, the components should be more reliable than is achievable by frequent maintenance of the equipment. In this case, the reliability of the system should be improved by applying a set of redundant elements. The most robust and flexible solution is the k-out-of-n configuration (”*koon*”) that consists of n components and fails if and only if at least a single k component fails. This structure has the benefit of having the opportunity to capacity among the k components and the quick repair of critical system failures that have been detected without stopping the operational process (online repair).

When $R_i^*(t)$ represents the reliability of the individual components, and k_i and n_i denote the numbers of operating and overall elements, respectively, the reliability of the redundant component is calculated based on the combination of the faultless operating paths given by the following equation:

$$R_i(t) = \sum_{l=k_i}^{n_i} \binom{n_i}{l} \left(R_i^*(t)\right)^l \left(1 - R_i^*(t)\right)^{n_i - l} \tag{3.5}$$

if $k_i = 1$ then just one component is sufficient enough to fulfil the requirements, so the above equation simplifies to:

$$R_i(t) = 1 - \left(1 - R_i^*(t)\right)^{n_i}. \tag{3.6}$$

By inserting the equation above into the path set-based polynomial reliability model of the whole system, the following model is produced which describes the redundancy of the entire system with $k_i, n_i, \forall\, i = 1, \ldots, N$ parameters:

$$R_0(t) = 1 - \Pi_{k=1}^{n_p} \left[1 - \Pi_{i \in \pi_k} \sum_{l=k_i}^{n_i} \binom{n_i}{l} \left(R_i^*(t)\right)^l \left(1 - R_i^*(t)\right)^{n_i - l}\right] \tag{3.7}$$

The goal of the multi-objective redundancy allocation problem is to maximise the reliability over a given lifespan $t = T$ and meanwhile minimise the additional investment cost related to the redundant components.

The multi-objective optimisation problem that consists of n objective functions, $f_i(\mathbf{x})$, $i = 1, \ldots, n$ can be presented in the form:

$$\min_{\mathbf{x} \in \mathcal{S}} \left\{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x})\right\} \tag{3.8}$$

subject to

$$\mathcal{S} = \{\mathbf{x} | g_k(\mathbf{x}) \leq 0,\ k = 1, 2, \ldots, m\} \tag{3.9}$$

where $\mathcal{S}$ denotes the set of feasible solutions defined by a set of $g_k(\mathbf{x}) \leq 0$ nonlinear constraints and $\mathbf{x}$ represents a decision vector that contains the decision variables as $\mathbf{x} = [\mathbf{k}\,,\ \mathbf{n}]$, where $\mathbf{k} = [k_1, \ldots, k_N]$ and $\mathbf{n} = [n_1, \ldots, n_N]$.

At least two objective functions should be defined to maximize the reliability function $f_1(\mathbf{x}) = 1 - R_0(\mathbf{x}, T)$, and minimize the installation (and maintenance) costs $f_2(\mathbf{x}) = C(\mathbf{x})$ of the redundant components as the number of installed units determines the investment cost, $C(\mathbf{x}) = \sum_{i=1}^{N} n_i ic_i$, where ic_i denotes the investment cost or the investment and maintenance cost-related weight of the i-th component.

The most often realised and standardised variants of *koon* are 1oo2, 2oo2 and 2oo3 (triple modular redundancy), where the number of operating units should be smaller than the number of installed units, i.e. the equation

$$1 \leq k_i \leq n_i \leq 3\,, \, \forall i = 1, \ldots N \tag{3.10}$$

should be fulfilled.

In order to obtain information about how the objectives conflict with each other and let safety engineers select according to the extracted trade-off between risk and reliability, the aim of the proposed optimisation-based sensitivity analysis is to identify a set of Pareto optimal solutions where a decision $\mathbf{x}_i$ is a Pareto optimal solution if both conditions are true:

- $\mathbf{x}_i$ is no worse than $\mathbf{x}_j$: $\forall_k f_k(\mathbf{x}_i) \leq f_k(\mathbf{x}_j)$
- $\mathbf{x}_i$ is strictly better than $\mathbf{x}_j$ in terms of at least one objective: $\exists k : f_k(\mathbf{x}_i) < f_k(\mathbf{x}_j)$

As the problem is NP-hard, recently several metaheuristic algorithms have been developed to obtain the Pareto fronts of complex optimisation problems. Among these, the most widely applied NSGA-II is utilised [24], which is a multiple-objective optimisation algorithm and its applicability will be demonstrated in the following section.

3.3.3 Case study

The applicability of the proposed methodology is demonstrated using a real-life case study related to the reforming reaction system at Sinopec Luoyang Petrochemical Plant [51] (see Figure 3.2). As high production losses and maintenance costs cannot be tolerated, a risk-based maintenance strategy has been developed to reduce economic risk due to unexpected failures. In contrast to Chapter 2, the case study is now being used for validating the time-dependent model, and thus, Weibull parameters are used.

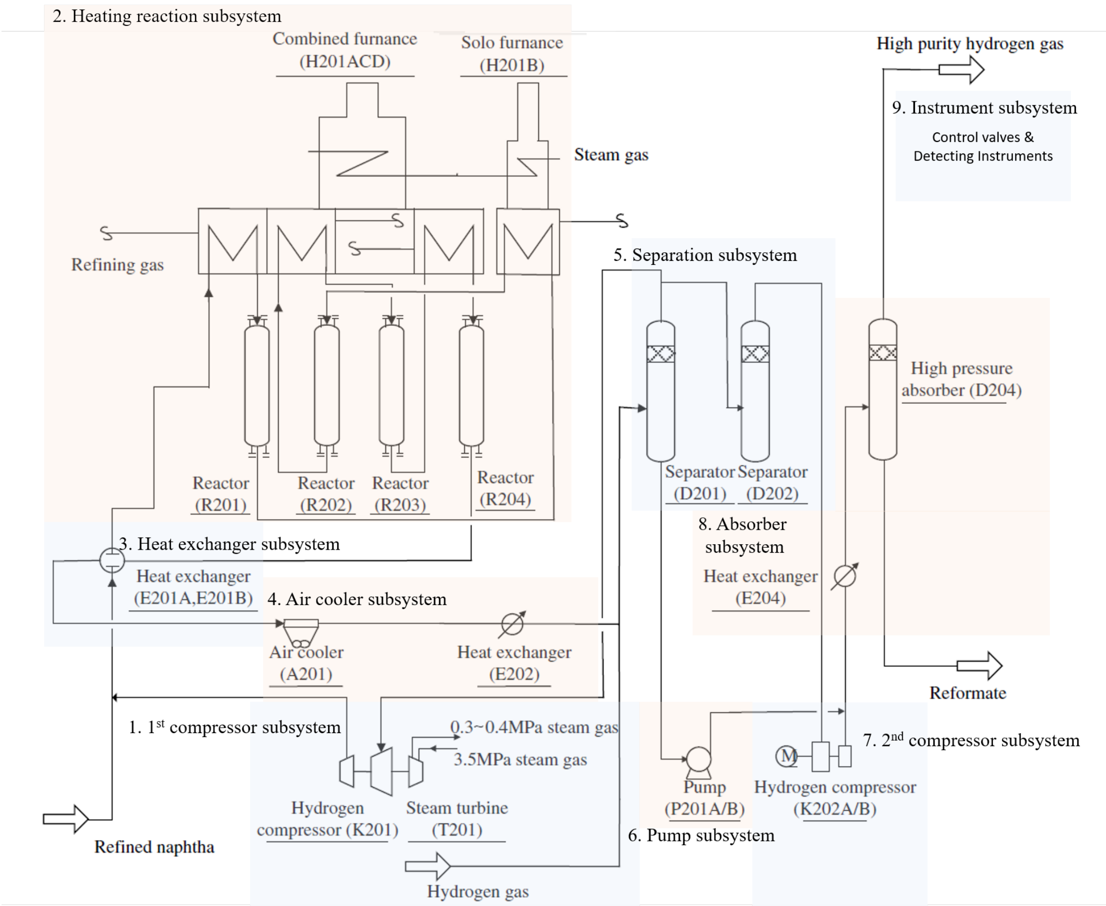

Figure 3.2: The nine subsystems of the studied reforming reaction system

Besides the application of a suitable maintenance methodology, the introduction of redundant processing units can decrease reliability-related risks. Thanks to the well-documented and realistic maintenance costs and production losses as well as the identified reliability models (shown in Tables 3.2 and 3.3), this risk-based maintenance problem can be extended to serve as an excellent demonstration of the proposed redundancy allocation method.

Table 3.2: The risk of subsystems is calculated based on the maintenance cost $MC_i = cfm_i + DT \cdot CV_i$ which consists of the fixed costs cfm_i (e.g. inspection, component replacement) and variable costs CV_i (e.g. costs of labour) associated with the downtime DT_i. The effect of the production loss $PL_i = DT_i \cdot PLPD_i$ calculated based on downtime DT_i and daily production loss $PLPD_i$ is also accounted by the risk, so $Risk_i = MC_i + PL_i$ [51]

#	Subsystem	Reliability ($P(e_i = 1)$)	cfm_i (\$)	DT_i (day)	CV_i (\$)	$PLPD_i$ (\$)
1	1^{st} compressor	0.4208	$2,173.9$	1.5	144.93	$43,478$
2	Heating-reaction	0.4011	$7,246.4$	5.0	289.86	$43,478$
3	Heat exchanger	0.6088	$2,898.6$	3.0	289.86	$43,478$
4	Cooler	0.6801	$1,449.3$	2.0	289.86	$43,478$
5	Separation	0.9907	$2,898.6$	4.0	289.86	$21,739$
6	Pump	0.5722	724.6	1.0	72.46	0
7	2^{nd} compressor	0.7874	$1,449.3$	1.0	144.93	0
8	Absorber	0.6984	$1,449.3$	4.0	144.93	$14,493$
9	Instrument	0.4141	724.6	1.0	72.46	0

Table 3.3: Parameters of the $F_i^*(t) = \exp^{-\left(\frac{t}{\beta_i}\right)^{\alpha_i}}$ Weibull probability distributions describing the reliability of the units in the reforming reaction system [51].

#	Subsystem	Equipment	Scale parameter β/month	Shape parameter α	Improvement factor ρ	Cumulative failure probability $F(t)$ (over 1 year)
1	1^{st} compressor	Steam turbine (T201) Hydrogen compressor (K201) Steam feed pipeline	19.001 17.711 120.000	2.713 1.895 1	0.822 1 N/A	0.24979 0.38011 0.09516
2	Heating-reaction	Reactors (R201 / R202 / R203 / R204) Combined furnace (H201 ABC) Solo furnace (H201B) Refining gas feed pipeline	40.181 18.842 20.774 150.000	3.154 1.833 2.147 1	0.897 0.917 0.815 N/A	0.02187 0.35426 0.26495 0.07688
3	Heat exchanger	Heat exchangers (E201A / E201B)	22.802	2.171	0.770	0.21977
4	Cooler	Air cooler (A201) Heat exchanger (E202)	40.574 19.546	2.835 2.129	1 1	0.03114 0.29807
5	Separation	Separators (D201/D202)	45.285	4.038	0.823	0.00468
6	Pump	Pumps (P201A / P201B)	12.329	1.772	0.604	0.24354
7	2^{nd} compressor	Hydrogen compressors (K202A / K202B)	15.068	2.307	0.727	0.11265
8	Absorber	High pressure absorber (D204) Heat exchanger (E204)	33.083 25.139	2.100 1.929	1 0.781	0.11209 0.21348
9	Instrument	Control valves&Detecting Instruments	N/A	N/A	N/A	0.58590

Instead of solving a process synthesis problem, in this study, the P-graph of the process was obtained based on the process flow diagram of the system (see Figure 3.3). As all the elements of the R and P sets of the P-graph are represented by

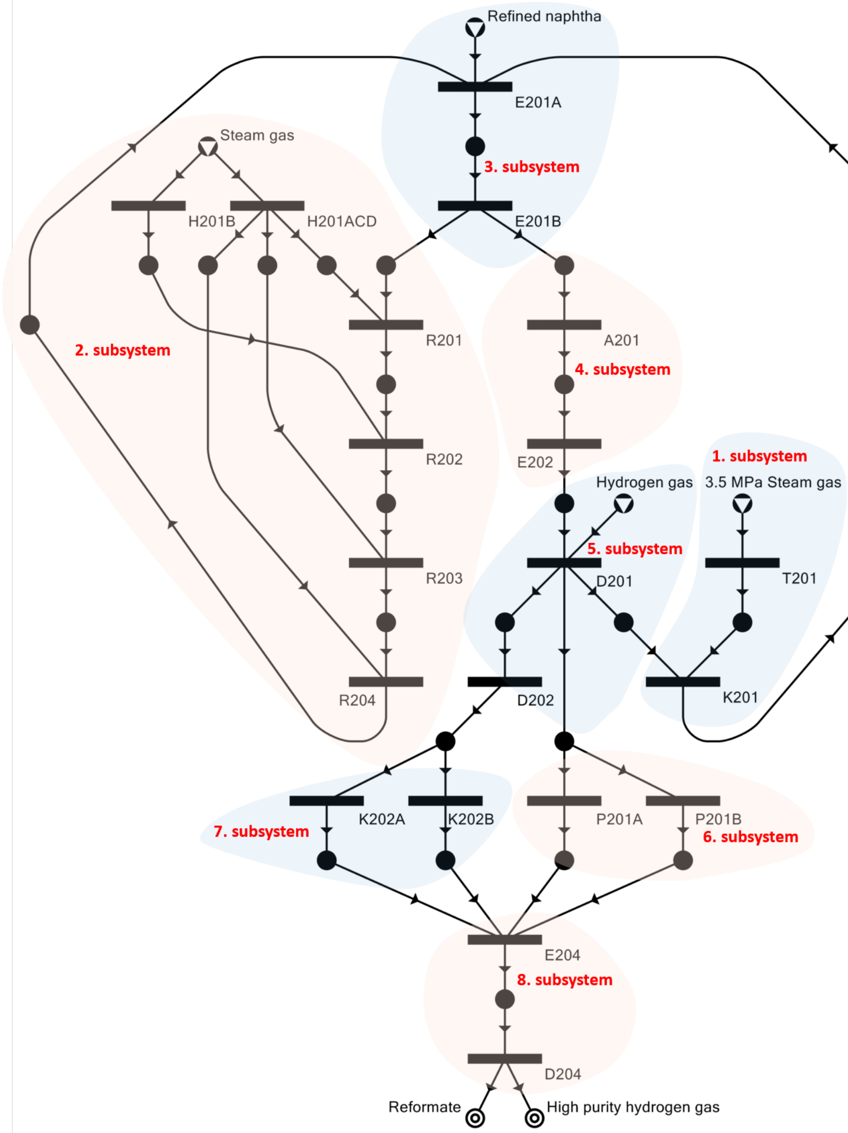

Figure 3.3: P-graph of the reforming reaction system highlighting the different subsystems

elementary reliability functions, with the help of the path set generation algorithm the fault tree of the process can also be generated. Given the strong dependency among the components, only one path set was identified which was decomposed according to the hierarchy of the technology (see Figure 3.4).

The failure probabilities of subsystems are calculated using the polynomial model (see Equation 3.5). The risks can be evaluated by multiplying the fault probabilities by the production losses due to downtimes as well as fixed and variable costs

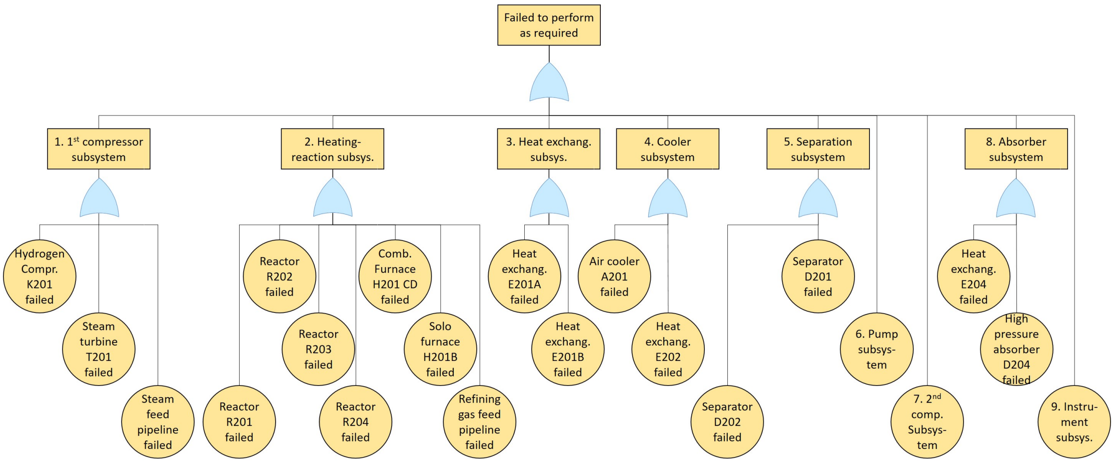

Figure 3.4: Fault tree of the reforming reaction system

of maintenance activities. As Table 3.2 in the appendix shows, the effects of failures and production losses are different in the subsystems, so the risk is calculated as $Risk = \sum_i (1 - R_i(T)) * L_i$, where $R_i(T)$ represents the reliability of the i-th subsystem at the end of the planning period and L_i stands for the loss when the i-th subsystem fails. According to this interpretation, the risk is the expected loss (expressed in \$) over the time period T.

The time-varying risks of equipment failures were calculated and summarised according to the hierarchy of the technology as is shown in Figure 3.5, 3.6, and 3.7.

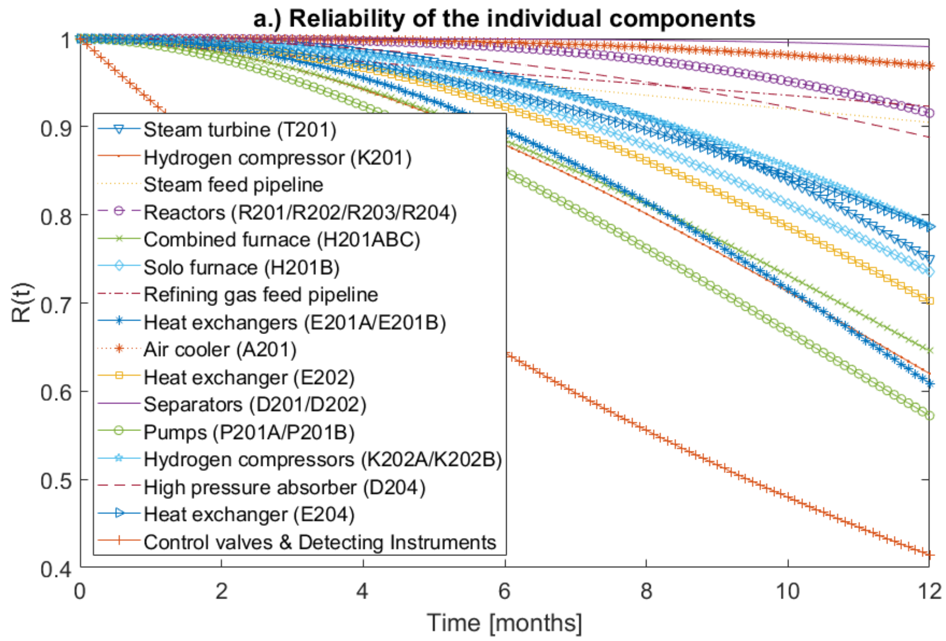

Figure 3.5: Reliability of the individual components

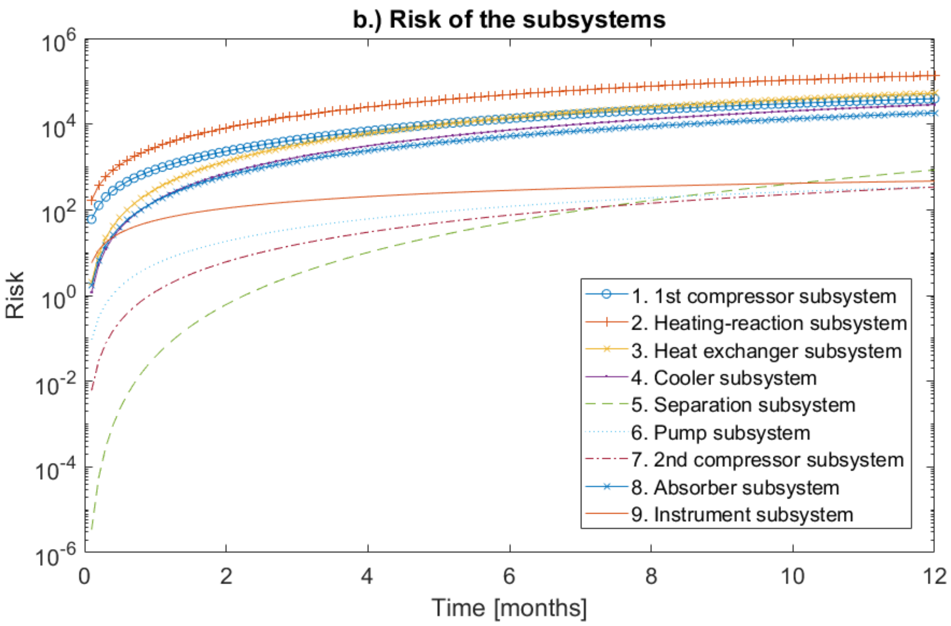

Figure 3.6: Risk of the subsystems

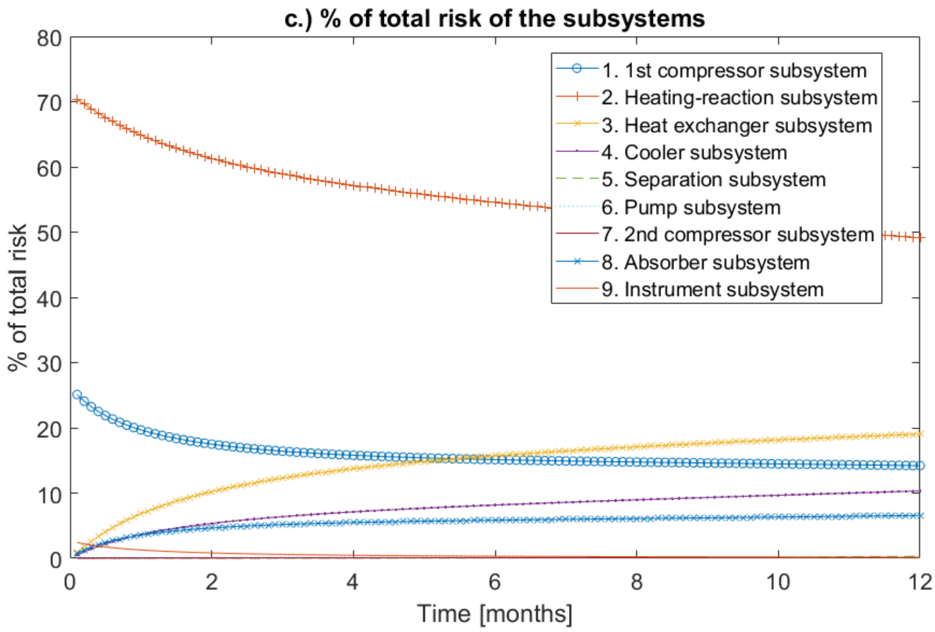

Figure 3.7: % of the total risk of the subsystems as a function of time

The result of this risk evaluation can be used to measure the importance of the units and subsystems. The Pareto diagram of this analysis is shown in Figure 3.8. The comparison between the risks after the first month and first year confirms the importance of the time-dependent analysis. The main conclusion from this plot is that the relative importance of the subsystems varies over time so time-variation should be taken into account not only during the design of maintenance periods but also in terms of redundancy allocation. Both short- and long-term aspects of reliability should be taken into account as it is necessary to ensure reliability over time intervals shorter than the maintenance period and also over more extended periods in process units in which the improvement factor of maintenance conducted is small.

The primary goal is to identify the safety-critical units and determine what kind of redundancy is worth applying to increase their reliability even when specific information about the cost of the units is not available. This challenge is handled by formalising the risk-based redundancy allocation problem as a multi-objective optimisation task that simultaneously minimises risk and the numbers of redundant units.

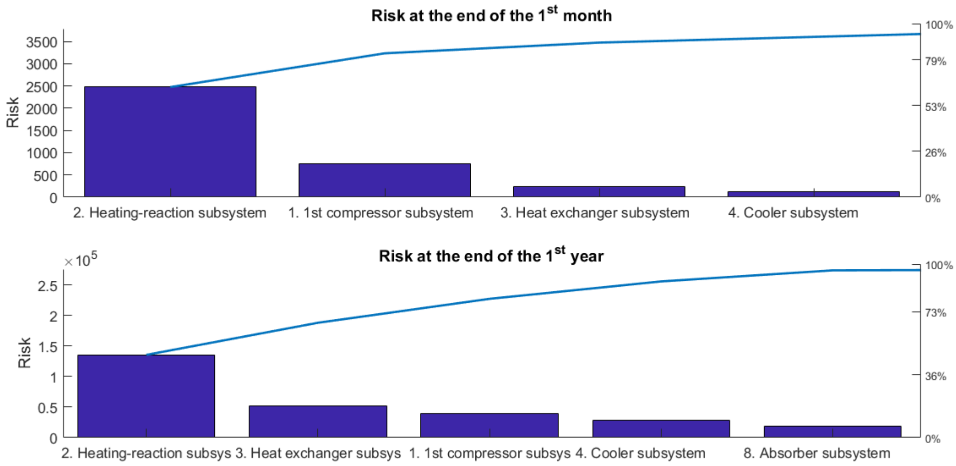

Figure 3.8: Pareto analysis of the risk (the expected loss in US $) after the a.) first month and b.) first year

It has to be noted that although this methodology should be primarily considered as a sensitivity analysis when much more accurate information about the capital costs of the redundant elements and the effect of their maintenance is proposed, optimisation can serve as an advanced design tool.

To determine the non-dominated set of optimal solutions, the widely applied NSGA-II was used. A detailed flowchart of the algorithm is shown in Figure 3.9. According to the nature and complexity of the mathematical model, a complex nonlinear optimisation problem is given, thus the global optimum is not guaranteed in all cases. As this genetic algorithm-based tool utilises a stochastic meta-heuristic search, ten independent runs were performed on a large population and generations, $N_{pop} = 100$ and $N_{gen} = 100$, respectively. The mutation, crossover rates, etc. parameters have been remained unchanged, default values have been applied. As is shown in Figure 3.10 the algorithm yielded consistent results when *koon*-type of redundancy was optimised when $k_{max} = 2$ and $n_{max} = 3$ and the sum of additional redundant elements was constrained as $\sum n_i \leq 10$. Note that in this case, the maximum number of units at a subsystem is at most 3, as it can be shown that greater redundancy is usually not economical [16].

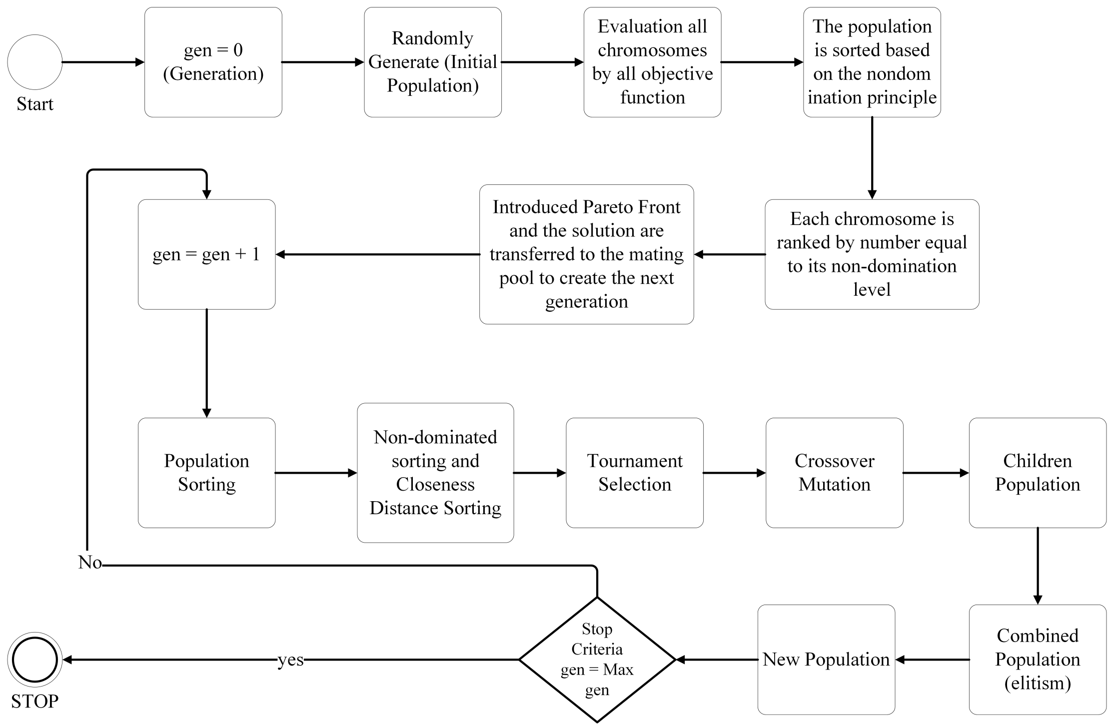

Figure 3.9: The flowchart of the utilised NSGA-II optimisation algorithm

As is shown Figure 3.10, the risk can be significantly reduced by adding redundant components. The main benefit of the proposed multi-objective optimisation-based sensitivity analysis is that it is unnecessary to define a detailed cost function for screening the redundancy strategies, the analysis of the resultant Pareto front already highlights how risk can be reduced by the utilisation of *koon* redundancy.

The importance of the units can be calculated based on how frequently they are selected as redundant elements in the Pareto fronts of the ten independent optimisations. The consistency of the selection is visualised in box plots that were also aggregated according to the hierarchy of the technology to evaluate the risk-improvement potential of the subsystems (see Figure 3.11). The selection-based importance of the individual units of the system is illustrated by the top part, while in the bottom part it can be seen that their importance is aggregated into the defined subsystems.

It should be noted that although the results are similar to the risk-based Pareto

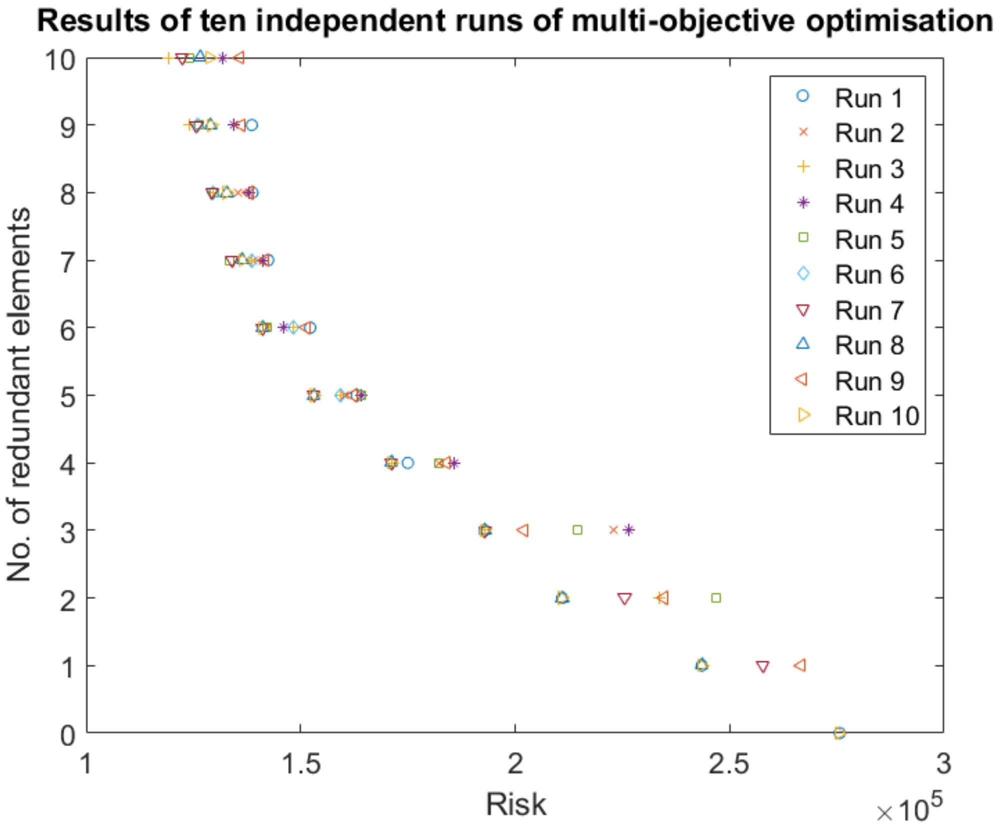

Figure 3.10: The results of ten independent runs of multi-objective optimisation-based redundancy allocation using NSGA-II. The similar Pareto fronts confirm the consistency of the meta-heuristic search. As can be seen, the increase in redundancy significantly decreases the risk (the expected loss in US $)

analysis presented in Figure 3.8, the proposed multi-objective optimisation-based analysis highlights the risk-improvement potential of the components and subsystems based on the prior knowledge of the safety and process engineers concerning the costs of making the units redundant.

When investment costs related to the building up of the redundant elements are taken into account; the results imply first that the reliability of Heat exchangers (E201A/E201B) in the third subsystem should be improved. Since the reliability of these units improved by only 77% following their maintenance (see the Improvement Factor in Table 3.2), it is worth investigating the economic benefits of the implementation of the suggested redundancy in more depth.

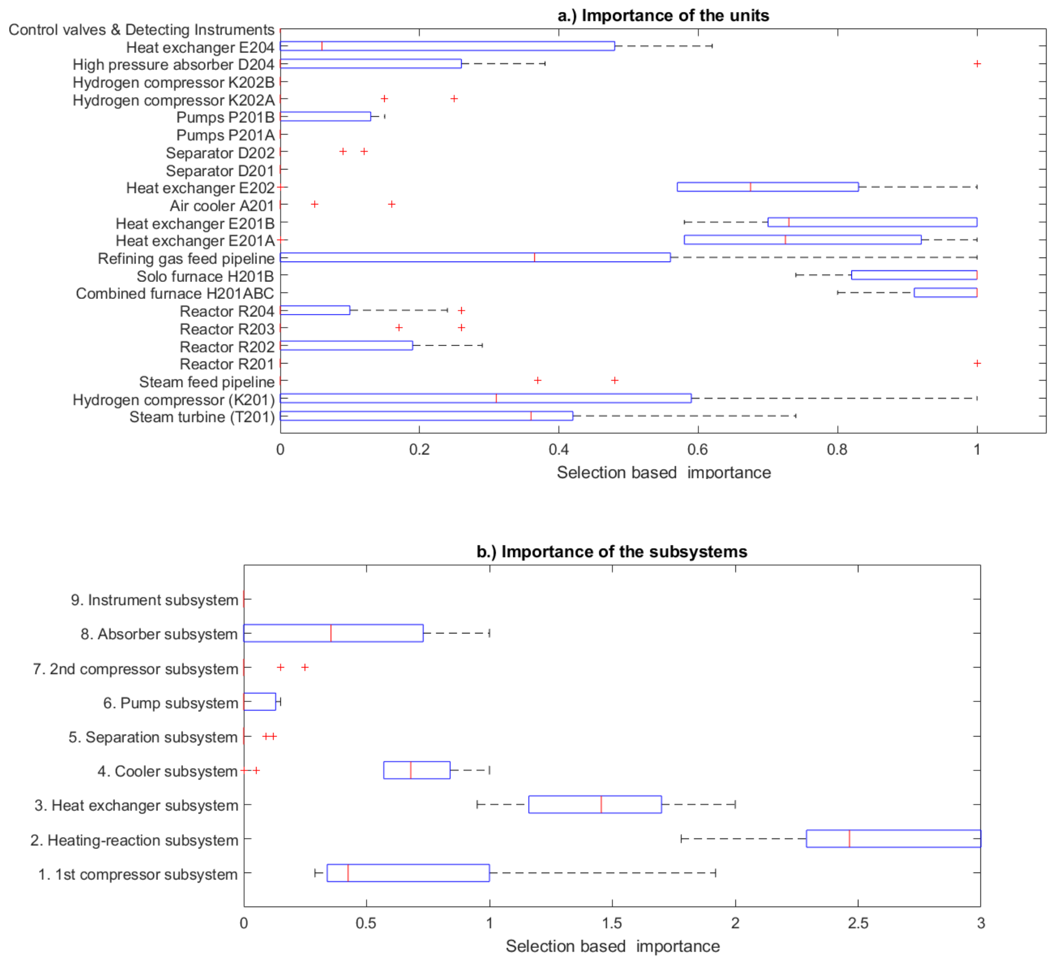

Figure 3.11: a.) The importance of the units and b.) subsystems is evaluated based on the results of the multi-objective optimisation. The frequency of the selection of the redundant elements is evaluated by box plots.

3.4 Major results and related publications

As P-graphs are widely used in terms of the optimisation of energy transformation as well as production systems and the optimisation of these systems should often handle risk-related aspects, the objective of the proposed research was to present a method for P-graph-based risk and reliability analysis.

It was determined that P-graphs also represent the logical dependencies between the availability of raw materials and the operating units, moreover, these models can be extended to incorporate additional logical conditions/dependencies in order that the path and cut sets of P-graphs represent the logical conditions of the operation of the whole process are ensured.

The path sets lend themselves to the extraction of reliability block diagrams and polynomial risk models from the P-graphs. It has been demonstrated that the hierarchy of the technology can be used to partition the optimisation model. This transparent representation is beneficial as it can be easily used to calculate of risks related to the malfunction of the subsystems and determine the risk-based evaluation of the importance of the process units.

The proposed approach has proven to be useful in terms of redundancy allocation formulated by multi-objective optimisation tasks. The methodology developed can be applied in all cases where the relationships between a given system and its subsystems can be formally described and the target of the investigations is in connection with reliability. These questions can of course be extended by further constraints and new optimization models can be derived accordingly. The applicability of this approach was validated by a case study related to a reforming reaction system. The results confirm that the main benefit of the proposed scheme is that even if detailed information concerning investment costs is unavailable, the method is still an excellent tool for the evaluation of the criticality of the components and comparison between different redundancy strategies.

According to these, the main contributions of the work were as follows:

- Overview of recent efforts concerning the P-graph based optimisation of energy systems and redundancy allocation in these processes which proves the importance of the research.
- Path-set based automatic extraction of polynomial risk models from P-graphs.
- Time-varying risk analysis and the importance with regard to the evaluation of the units of P-graphs.
- Statistical evaluation of a set of independent multi-objective optimisation

runs is used for the evaluation of the importance of the safety-criticality of the components and subsystems.

- A case study based on industrial data demonstrated that the methodology is applicable to risk-based resource allocation.
- Comparison with classical risk-priority based Pareto analysis shows the benefits of the proposed approach.

The incorporation of the proposed methodology in the P-graph based design of energy systems is not only recommended but the improvement in the reliability of safety-critical systems in retrofitting studies should also be taken into account as an objective. The methodology is flexible as P-graphs are excellent for the modelling of process systems so with the help of the proposed method it can serve as an interface between process flow diagrams and polynomial risk models.

The importance of taking into account the time-variance of risks in terms of the allocation of redundant components was also highlighted by the results.

Related publications

- Süle Z., Baumgartner J., Dörg Gy., Abonyi J., 2019, P-graph-based multi-objective risk analysis and redundancy allocation in safety-critical energy systems, Energy, 179, 989-1003, DOI:10.1016/j.energy.2019.05.043, IF: 6.947, Rank: **D1**
- Süle Z., Baumgartner J., Abonyi J., 2018, Reliability - redundancy allocation in process graphs, Chemical Engineering Transactions, 70, 991-996, DOI:10.3303/CET1870166, Rank: **Q3**

3.5 Thesis 2

I have generalized the methodology of P-graph-based reliability calculation to the time-dependent case and k-out-of-n ($k$oo$n$) configurations. The developed model can be easily applied to calculate risks related to the malfunction of the subsystems and determine the risk-based importance of the process units.

- I have defined cut set-based metrics for the identification of the critical elements in a system.
- I have developed a multi-objective formulation of the koon redundancy allocation problem, where time dependent reliability of a given system, the degree of the redundancy, and the related costs can be optimized.

Chapter 4

Test sequence optimisation by survival analysis

Testing is an indispensable process for ensuring product quality in production systems. Reducing the time and cost spent on testing meanwhile minimising the risk of not detecting faults is an essential problem of process engineering. In the following, the method for optimisation of complex testing processes consisting of independent test steps is considered. Survival analysis based models of the elementary test are developed to efficiently combine the time-dependent outcome of the tests and costs related to the operation of the testing system. A mixed integer non-linear programming (MINLP) model is proposed to formalize how the total testing cost depends on the sequence and the parameters of the elementary test steps. To provide an efficient formalization of the scheduling problem and avoid difficulties due to the relaxation of the integer variables the MINLP model is considered as a P-graph representation based process network synthesis problem. A realistic case study taken from computer manufacturing demonstrates the applicability of the methodology. With the application of the optimal test times and sequence provided by the SCIP (Solving Constraint Integer Programs) solver 0.1–5% of the testing cost can be saved.

4.1 Introduction

In the chapter I am focusing on the optimisation of modular, replaceable unit based production systems. The modular production involves distributors and suppliers in the manufacturing process [87], which increased integration improves responsiveness to customers and efficiency [92]. Industry 4.0 is a strategic approach to design optimal production flows by leveraging the interconnectivity to reach the goal of intelligent, resilient and self-adaptable manufacturing systems [67].

Testing is an indispensable process in production systems. Usually, the almost independently operating modules of modular products are tested as a sequence of independent test steps related to testing the independent units. The primary focus of the test steps is to identify the faulty modules rather than the individual faults within the modules. When a test step fails, the defective unit will be removed and replaced, and the product is retested.

The aim is to determine the test sequence which minimizes the expected cost.

The sequencing problem initially focused to the optimisation of the diagnostic and fault isolation functions of electronic products. Troubleshooting built-in test sequence optimisation is a classical problem in the design of automatic systems [86].

The diagnosis is often integrated with two types of repair: Type 1 repair wherein a module is repaired after complete diagnosis, and a Type 2 repair where a module suspected to be faulty is replaced after partial diagnosis. For systems during operation the integration of these repair strategies into the problem of which tests must be executed in what sequence was already solved by Pattipati [78].

Test sequencing problems during manufacturing require a different approach than test sequencing problems during operation [18]. Contrary to previous works originated from the analysis of fault probabilities, we aim to build a detailed cost function of the testing procedure and give a sophisticated solution to the problem.

Test prioritization algorithms for fault localization are based the diagnostic information gain per test to enhance the rate of fault detection [44].

The traditional test-sequencing problem includes asymmetrical tests where the next test to execute depends on the results of previous tests. Hence, the test-sequencing problem can naturally be formulated as a decision tree construction problem, whose solution is known to be NP-hard [62]. In this chapter, I highlight that in manufacturing although we have to test all the components, the total costs of the sequence depends on the test sequence, since the number of the tested products is influenced by the results of the previous tests.

In most of the cases, the tests have fixed time interval. The decision of when to stop testing is often difficult to make because less testing may leave critical faults in the system, while more testing increases the costs and the time-to-market. A risk-based stopping criterion of deciding when to stop testing has been introduced for test sequencing in [17].

The aim is to build a complete test time cost and risk cost model based on the survival analysis of the historical data of the test process and use the resulted model for rescheduling of the test sequence.

Although I study a different problem than maintenance optimisation, at the development of the model, several elements can be utilized from this field. In the context of risk-based maintanance optimisation failure history and lifetime distribution function based optimisation of inspection periods was already examined in 1972 by Zacks and Fenske [103]. Detailed optimisation models of periodically inspected preventively maintained units take into account finite repair and maintenance durations as well as costs due to testing, repair, maintenance and lost production [96]. Repair-time limit replacement problem with the imperfect repair was also studied in [26].To predict the number of spare components required to maximize the availability of a system a non-linear integer programming problem was defined [61]. The optimisation model uses exponential, gamma, normal and Weibull distributions to represent how the probability of failures vary in time. The risk model plays an essential role in these optimisation problems. In advanced solutions to describe the failure rate, Cox's proportional hazards models and Weibull hazard functions with time-dependent stochastic covariates are used, and the parameters of the hazard functions were estimated using maximum-likelihood and

Quasi-Newton methods [8].

In this chapter, a risk-based test sequencing optimisation algorithm is developed based on the techniques learned from risk-based maintenance optimisation. I apply of survival analysis and hazard functions to formalize a sophisticated test cost model; we optimise the lengths of the tests steps and formalize the integrated sequencing task as a Mixed Integer Nonlinear Problem.

The mathematical model of the test sequencing optimisation problem can be constructed as a traditional scheduling problem formulated as standard mixed integer mathematical programming. This formulation represents the ordering of the tests as a set of constraints defined on integer variables. Problem specific simplifications of the testing process can hardly taken into account in such models, thus the optimisation process can take a long time for a large number of test steps.

The fundamental idea is that the benefits of the algorithmic superstructure generation and P-graph framework are used initially introduced for process network synthesis PNS problems [38] to generate a mathematical model which exploits the structural properties of the testing process.

The analogy between the separation network synthesis and test sequencing optimisation problems is that items failed in a test step are separated from items which passed the test. Separation network synthesis problems (SNS) aim is to design an optimal separation structure that separates the components of input streams into outlet streams of specific composition. The algorithmic generation of rigorous super-structure that includes this optimal structure is an efficient approach to solve these problems [59]. An algorithm for the generation of a problem-specific reduced super-structure with minimal complexity has been developed and applied in [48, 49]. The main contribution of this chapter is that the detailed cost model of a test sequencing problem is formalised and the SNS based representation to generate its parsimonious MILP model is used. The demonstration of the applicability of this approach is made by a realistic case study taken from computer manufacturing.

4.2 Notations

The functions, variables and parameters required for the test sequence optimisation problem are listed below:

Functions:

$C^i(t)$	-	Total cost function of test step i
$C_f^i(t)$	-	Fix and amortization cost function of test step i
$C_p^i(t)$	-	Proportional cost function of test step i
$C_r^i(t)$	-	Repair cost function of test step i
$C_w^i(t)$	-	Warranty cost function of test step i
$S^i(t)$	-	Survival function of test step i
$W^i(t)$	-	Weibull distribution function of test step i

Variables:

π	-	Vector representing the order of test steps
t	-	Variable representing time
t_i	-	Length of the i^{th} test step
$\bar{t}$	-	Vector of the times of activities
x_i	-	Volume of activity i
$\bar{x}$	-	Vector of the volumes of activities
y_i	-	Existential (binary) variable representing the status of activity i (works or not)
$\bar{y}$	-	Vector of existential (binary) variables representing the status of activities
Z	-	$n \times n$ matrix representing test orders

Parameters:

c_f^i	-	Constant fix and amortization cost of test step i for one item
c_p^i	-	Constant proportional cost of test step i for one item
c_r^i	-	Constant repair cost of test step i for one item
c_w^i	-	Constant warranty cost of test step i for one item
Exl	-	set representing the mutually exclusive activities
e_i	-	i^{th} unit vector

k_i	-	Shape parameter of Weibull distribution $W^i(t)$
λ_i	-	Scale parameter of Weibull distribution $W^i(t)$
L_{p_j}	-	Lower bound for m_j final target (product)
N	-	Number of test steps
N^i_{in}	-	Number of items involved in test step i
N_{in}	-	Number of items entering the testing process
M	-	Set of entities in PNS problem
$\wp(M)$	-	Cartesian product of set M
$\bar{M}$	-	Sufficiently large number
P	-	Set of product in PNS problem
R	-	Set of entities in PNS problem
$ratio_{ji}$	-	Function representing the difference between the production and
		consumption rate of entity m_j by activity o_i
O	-	Set of operating units in PNS problem
O_1	-	Set of operating units representing test steps in the process
O_2	-	Set of operating units representing logical (not test steps) activities in the process
T^i_{max}	-	Maximum length of test step i
U_{c_j}	-	Upper bound for m_j resource

4.3 Formulation of the test sequence optimisation problem

The studied test process consists of $i = 1, \ldots, N$ test steps. The time dependent costs of the test steps are represented by the $C^i(t)$ functions. The costs are linearly proportional to the number of items passing through the test steps, $C^i(t)N^i_{in}$, where N^i_{in} represents the number of items involved in the test step i .

The studied test process is linear. The order of the test steps is represented by a

π vector, as $i = \pi_j$ denotes that the jth element of the sequence is the ith test step.

The applied $S^i(t)$ survival functions are represented by $W^i(t, k_i, \lambda_i)$ Weibull distribution to determine the ratio of items that successfully pass a test step i

$$S^i(t) = 1 - W^i(t, k_i, \lambda_i) = e^{-(t/\lambda_i)^{k_i}}, \text{ for } t \geq 0. \tag{4.1}$$

Items failed in a test step are not included in the further test steps, so

$$N_{in}^{\pi_{j+1}} = N_{in}^{\pi_j} S^{\pi_j}(t_{\pi_j}) = N_{in} \prod_{l=1}^{j} S^{\pi_l}(t_{\pi_l}) \tag{4.2}$$

where N_{in} is the number of items entering the testing process and t_{π_j} represents the length of the π_jth element of the sequential test process.

Thus, the total cost of the testing process can be formalized as follows:

$$\min \sum_{i=1}^{N} C^i(t_i) N_{in}^i = \sum_{i=1}^{N} C^i(t_i) N_{in}^{\pi_{i-1}} S^{\pi_{i-1}}(t_{\pi_{i-1}}), \tag{4.3}$$

where $N_{in}^0 \equiv N_{in}, \pi_0 \equiv 0$ and $S^0(t_0) \equiv 1$.

In more compact and transparent form the optimisation problem is defined as determining the order of the test indexes represented by the π vector and the t_π vector that consists of the time lengths of the test steps:

$$\min_{\pi, t_\pi} C(\pi, t_\pi) = N_{in} \sum_{j=1}^{N} \left(C^{\pi_j}(t_{\pi_j}) \prod_{l=1}^{j-1} S^{\pi_l}(t_{\pi_l}) \right) \tag{4.4}$$

The $C^i(t)$ function of the test steps is introduced to represent the fix, proportional, repair and warranty cost elements:

$$C^i(t) = C_f^i(t) + C_p^i(t) + C_r^i(t) + C_w^i(t) \tag{4.5}$$

where C_f^i stands for the fix and amortization costs:

$$C_f^i(t) = c_f^i t \tag{4.6}$$

and C_p^i the proportional cost elements:

$$C_p^i(t_i) = c_p^i \int_0^{t_i} S^i(t)\mathrm{d}t\,. \tag{4.7}$$

The parameters c_f^i and c_p^i give the constant fix and amortization as well as the proportional costs of test step i for one item.

Wrong items discovered during the testing process will be repaired in a later working phase. The $1 - S^i(t)$ function gives the ratio of the failures at time t, thus the repair cost function is

$$C_r^i(t_i) = c_r^i \left(1 - S^i(t_i)\right), \tag{4.8}$$

where c_r^i defines the repair cost for one item.

Some failures remain hidden despite of testing steps, thus these items must be repaired during the warranty period. The last cost function represents this cost element, where c_w^i shows the warranty cost for one item and T_{max}^i is the maximum length of the test step i:

$$C_w^i(t) = c_w^i(S^i(t) - S^i(T_{max}^i)). \tag{4.9}$$

The goal of the test sequence optimisation problem is to minimise the overall cost representing Eq. 4.4 with the constraints of the mathematical model come from the sequence search problem, where each test must be executed exactly once in a given order. Thus, we get such a scheduling problem, where the objective function can be evaluated only if a feasible test order is available since the binary variables which determine the optimal order can not be relaxed during the optimisation steps.

4.4 Test sequence optimisation as a process network synthesis problem

In a test sequence optimisation problem the optimal test order and test durations have to be given, where the constraints guarantee the right permutation of test steps and the minimisation steps give the optimal test times. The general mathematical model can be formalised as a Mixed Integer Nonlinear Programming problem:

$$\begin{aligned}
\min_{\pi,t_\pi} \quad C(\pi,t_\pi) = \quad & N_{in}\sum_{j=1}^{N}\left(C^{\pi_j}(t_{\pi_j})\prod_{l=1}^{j-1}S^{\pi_l}(t_{\pi_l})\right) \\
\text{s. t.} \quad & \text{vector } \pi \text{ represents the permutation of } N \text{ test steps} \\
& \text{where} \\
& t_i \geq 0, 1 \leq \pi_i \leq N \text{ integer}, \\
& i = 1, 2, \ldots, N
\end{aligned} \tag{4.10}$$

The objective function of the problem (4.10) can be evaluated only if feasible order of test steps is known.

Although the problem to be solved can be described by several optimisation models, in my dissertation, the test-sequence optimisation task is investigated using process network synthesis-based formalisation. One of the advantages of the approach is that the mathematical model can be easily generated as the structure of the task changes, and the P-graph framework provides several algorithms for optimisation and analysis of the task that can even produce all feasible solutions.

Considering the test sequence optimisation problem as a process network synthesis (PNS) task, the relaxation of the binary variables does not cause any difficulty.

The goal of a process network synthesis is to create products from raw materials through various transformations (e.g., activities, physical reactions, etc.). The aim of this chapter is to adapt the process network synthesis problem to the test sequence optimisation task.

Each test step in the test sequence optimisation problem is represented by operating unit, which can be characterized by some cost parameters such as fix, proportional, repair and warranty costs. The total cost of a test step depends on the entering number of items and the length of the test thereby determined the number of passed and failed tests. Non-negative continuous variable t_i shows the duration of test step i, and x_i non-negative continuous variable gives the quantity of produced items by the operating unit in test step i.

The initial structure or superstructure involves each candidate N test represented by operating units, as well as, the $N(N-1)$ potential changeovers from each test to any other tests. Since test executions are not parallel, but sequential, each test i_1 is followed by at most one test i_2 as the forthcoming test. Consequently, changeovers from test i_1 to any other test i_2 where $i_2 = 1, \ldots, N, i_1 \neq i_2$ are mutually exclusive, i.e., at most one of changeovers $\forall i_1 \in N : \{ch_{i_1,i_2} : i_2 = 1, \ldots, N, i_1 \neq i_2\}$ can be included in a feasible test process. Thus, the sum of the corresponding existence variables does not exceed 1:

$$\forall i_1 \in N : \sum_{i_2 \in \{1,\ldots,N\}, i_1 \neq i_2} y_{ch_{i_1,i_2}} \leq 1 \tag{4.11}$$

Note that the binary variable y_i does not appear in the objective function directly, but its value impacts the capacity of the operating unit i: $x_i \leq \bar{M} y_i$, where $\bar{M}$ is a sufficiently large number, and $y_i = 1$ if and only if, the assigned operating unit works.

Consequently, the objective function of the optimisation problem can be given as

$$\begin{aligned} \min \sum_{i=1}^{N} \Bigg[c_f^i t_i + \Bigg(c_p^i \cdot \int_0^{t_i} S^i(t) \mathrm{d}t + \\ + c_r^i \cdot \big(1 - S^i(t_i)\big) + c_w^i \cdot \big(S^i(t_i) - S^i(T_{max}^i)\big)\Big) \cdot x_i \Big] \end{aligned} \tag{4.12}$$

While operating units are characterized by capacity, cost parameters and functions, the material type nodes are characterised by mass-balance constraints. Some quantitative limits are given for raw materials and products:

- the lower bounds L_{p_j} for each m_j final target:

$$L_{p_j} = \begin{cases} > 0, & \forall m_j \in P \\ 0, & \text{otherwise} \end{cases}$$

- the upper bounds U_{c_j} for each m_j resource:

$$U_{c_j} = \begin{cases} > 0, & \forall m_j \in R \\ 0, & \text{otherwise} \end{cases}$$

In the P-graph describing a testing process there is only one raw material which represents the computers waiting for testing. The activities linked to the raw material are mutually exclusive, and depicted all the activities as operating units. In the solution structure one activity must be selected among them which determines the first test step. Further parts of the graph describe the test steps and the next possible test steps for each of them. All the test step activity nodes have three outputs: the first shows the number of failed items, the second one is an artificial node with a positive lower bound, which forces the operation of the activity, and the last node represents the items passed the test.

Fig. 4.1 illustrates a structure with three test steps. Using the labels of the figure:

$$R = \{m_1\}, P = \{m_6, m_9, m_{12}, m_{15}\}, M = \{m_1, m_2, \dots, m_{15}\},$$

$$O = \{O_1, O_2, \dots, O_{18}\},$$

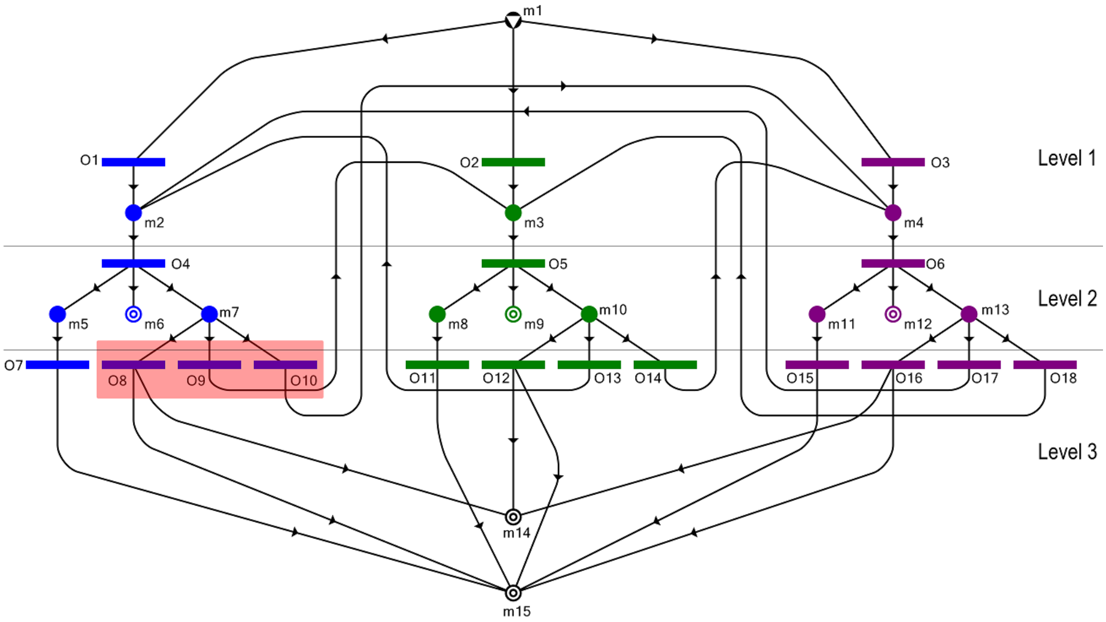

Figure 4.1: Sample P-graph describing a three test steps case.

$$Exl = \{\{O_1, O_2, O_3\}, \{O_8, O_9, O_{10}\}, \{O_{12}, O_{13}, O_{14}\}, \{O_{16}, O_{17}, O_{18}\}\}.$$

Level 1 in the P-graph helps to identify the first test step of the process, therefore only one activity can be chosen from the set $\{O_1, O_2, O_3\}$. The graph elements on Level 2 give the test steps with their cost functions. All the operating units will be part of the solution structure, since all tests must be executed. Finally, the elements of Level 3 help to give a right order of test steps, since structurally all possible connections are represented in the graph. The output of O_7, O_{11}, and O_{15} activities at Level 3 represent the failed items in test step 1, 2, and 3; exactly one activity is chosen from the set $\{O_8, O_{12}, O_{16}\}$, that one, which belongs to the last step element. The O_9 and O_{10}, the O_{13} and O_{14}, as well as the O_{17} and O_{18} give the next test steps. For example, if the test order is 3, 2 and 1, then O_{18}, O_{13} and O_8 activities are in the solution structure.

Formally, in the optimisation problem let M be the set of materials, O the set of activities, $\bar{x}$ the vector of the volumes of activities, $\bar{y}$ the vector of the existence variables to the activities, $\bar{t}$ the vector of the times of activities, and set

Exl represents the mutually exclusive activities. Relations between entities and activities are described by function $ratio_{ji}(t)$ which gives the difference between the production and consumption rate of entity m_j by activity o_i.

Let O split into two parts: $O = O_1 \cup O_2$, where $O_1 \cap O_2 = \emptyset$, O_1 contains those operating units which represent test steps in the process, while $O_2 = O \setminus O_1$. In this special PNS problem the set R contains only one element, since there is one starting point of the process.

The aim is to satisfy the following constraints and let z be minimal:

$$\forall o_i = (\alpha_i, \beta_i) \in O_1 : m_j \in \alpha_i \leftrightarrow ratio_{ji} = -1 \text{ and } m_j \in \beta_i \leftrightarrow ratio_{ji} > 0 \tag{4.13}$$

$$\forall o_i = (\alpha_i, \beta_i) \in O_2 : m_j \in \alpha_i \leftrightarrow ratio_{ji} = -1 \text{ and } m_j \in \beta_i \leftrightarrow ratio_{ji} = 1 \tag{4.14}$$

$$\bar{x} = [x_1, x_2, \dots, x_{|O|}]^T, \bar{y} = [y_1, y_2, \dots, y_{|O|}]^T \text{ and } \bar{t} = [t_1, t_2, \dots, t_{|O_1|}]^T \tag{4.15}$$

$$\forall x_i \geq 0, \forall y_i \in \{0, 1\}, \text{ and } \forall t_k \geq 0, i = 1, \dots, |O|, k = 1, \dots, |O_1| \tag{4.16}$$

$$m_j \in R : -U_{c_j} \leq \sum_{o_i \in O_2} ratio_{ji} x_i \leq 0 \tag{4.17}$$

$$\forall m_j \in P : L_{p_j} \leq \sum_{o_i \in O_2} ratio_{ji} x_i \tag{4.18}$$

$$\forall m_j \in M \setminus P \setminus R : \sum_{o_i \in O} ratio_{ji} x_i = 0 \tag{4.19}$$

$$\forall exl \in Exl : \sum_{o_i \in exl} y_i = 1 \tag{4.20}$$

$$\forall o_i \in O : x_i \leq \bar{M} y_i \tag{4.21}$$

$$z = \sum_{o_i \in O_1} \Bigg[c_f^i t_i + \Bigg(c_p^i \cdot \int_0^{t_i} S^i(t)\mathrm{d}t + \\ + c_r^i \cdot \big(1 - S^i(t_i)\big) + c_w^i \cdot \big(S^i(t_i) - S^i(T_{max}^i)\big)\Big) \cdot x_i \Big] \tag{4.22}$$

In the mathematical model Eqs. (4.13)-(4.14) show the different ratios between materials and activities; the output ratios of the material type nodes are equal to -1, while the input ratios depend on the type of connected activity. The positive $ratio_{ji}$ values in Eq. (4.14) are as follows: for each activity $o_i \in O_1$ has three outputs with the following properties:

- output m_{1i} represents the failed items with $ratio_{1i} = 1 - S^i(t_i)$.
- output m_{2i} ensures that the activity i works with $ratio_{2i} = L_{p_{2i}} > 0$.
- output m_{3i} gives the successfully passed items with $ratio_{3i} = S^i(t_i)$.

Eq. (4.17) gives the upper bound to the entering number of items, while (4.18) fixes the required leaving number of items at different points in the process. The (4.19) expresses the law of mass-balance, e.g. the entering number and the leaving number of items are equal at any intermediate point of the process. Due to the nature of the given scheduling task, some activities can not be performed concurrently; the set Exl contains these exclusions which are represented in Eq.(4.20) by the sum of binary variables.

4.5 Case study

To demonstrate the applicability of the proposed methodology an illustrative example is presented related to the optimisation of the sequence of functional tests of computers in a computer assembling process. In this example $N_{in} = 20,000$ computers are tested in a sequence of 12 test steps detailed in Table 4.2.

As discussed in the previous section, the failure events are characterised by survival functions which are derived from Weibull distribution. The Weibull parameters determine the failure rates of the test steps with parameters k_i and λ_i, and the cost properties are characterized using the parameters c^i_f, c^i_p, c^i_r, and c^i_w.

Before solving the general optimisation problem, some specific types of test sequence problem will be described.

First, consider that case when the costs of each test step are equal but the reliabil-

Table 4.2: Description of the functional test steps

#	Name	Description
#1	Sleep	Testing the PC can resume normal operation after sleeping for an extended amount of time
#2	Hibernation	Testing the operating system's ability to hibernate and the recovery from hibernation
#3	Restart	Testing the PC's ability to successfully restart the PC and the OS
#4	OS	Testing the operating system whether it can boot up or not
#5	HDD	Testing the HDD for bad sectors
#6	MemCheck	Running a memory checker to find faulty memory slots or RAM
#7	VGA	Testing the VGA if it sends out the display data properly
#8	USB	Testing of the USB ports with preinstalled USB simulators
#9	BIOS	Performing a BIOS power-on self-test (POST)
#10	CPU	Checking the temperature of the CPU under different load levels
#11	Power	Testing of the supplied voltage levels
#12	Stress	Testing the computer with multiple tasks at once

ities are different. Formally, providing that $C^1(t_1) = C^2(t_2) = \ldots = C^N(t_N) = C$, the objective function can be simplified to the following form, while the constraints are unchanged:

$$\min_{\pi, t_\pi} N_{in} C \sum_{j=1}^{N} \left(\prod_{l=1}^{j-1} S^{\pi_l}(t_{\pi_l}) \right) \tag{4.23}$$

It can easily be verified that the Eq. (4.23) is minimal when $S^{\pi_i}(t_{\pi_i}) \leq S^{\pi_j}(t_{\pi_j})$ for all $i \leq j$, where $i, j \in \{1, 2, \ldots, N\}$, i.e., the optimal solution is given by ascending order of reliability of test steps.

As an analogy to the former case let the reliabilities be the same while the costs be different. Formally, providing that $S^1(t_1) = S^2(t_2) = \ldots = S^N(t_N) = p$, the objective function is the following:

$$\min_{\pi, t_\pi} N_{in} \sum_{j=1}^{N} \left(C^{\pi_j}(t_{\pi_j}) p^{j-1} \right) \tag{4.24}$$

It can be seen that Eq. (4.24) is minimal if $C^{\pi_i}(t_{\pi_i}) \leq C^{\pi_j}(t_{\pi_j})$ for all $i \leq j$, where $i, j \in \{1, 2, \ldots, N\}$, i.e. the cheapest step is the first in the sequence and the others follow in ascending order.

The parameters of the model and the survival functions can be determined based on log files of testing processes by fitting the parameters of the survival functions to the available data sets. Table 4.3 summarises the parameters of the problem: the qualities of test steps are characterised by survival functions with parameters λ_i and k_i. The survival functions describing each test step are represented in Figure 4.2.

Table 4.3: The values of parameters in the optimisation problem

Step's name	λ_i	k_i	$c_f^i(\$)$	$c_p^i(\$)$	$c_r^i(\$)$	$c_w^i(\$)$
#1: Sleep	5×10^8	0.5	4.5×10^{-9}	3×10^{-5}	10	100
#2: Hibernation	7.8×10^{10}		1.5×10^{-10}	2×10^{-6}	9	91
#3: Restart	9.9×10^9		1.5×10^{-9}	10^{-5}	7	170
#4: OS	10^{10}		1.5×10^{-10}	10^{-5}	8	117
#5: HDD	2.9×10^{11}		4×10^{-10}	2×10^{-6}	9	163
#6: MemCheck	1.9×10^8		10^{-9}	6×10^{-5}	7	87
#7: VGA	7.3×10^7		10^{-9}	2×10^{-4}	14	146
#8: USB	1.6×10^9		5×10^{-9}	9×10^5	8	139
#9: BIOS	7.4×10^7		10^{-10}	10^{-4}	11	133
#10: CPU	2.7×10^9		5×10^{-10}	3×10^{-5}	11	128
#11: Power	2×10^{12}		1.5×10^{-10}	6×10^{-7}	6	101
#12: Stress	2.7×10^8		2×10^{-9}	6×10^{-5}	6	152

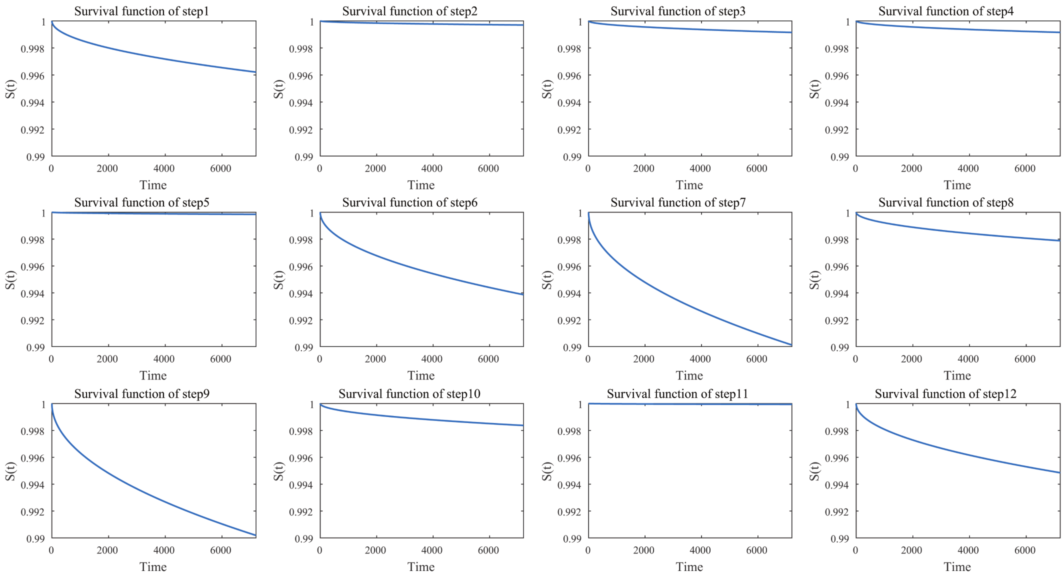

Figure 4.2: Survival functions of each test step

The parameters c^i_f, c^i_p, c^i_r, and c^i_w determine the characteristics of unit cost functions which are shown in Figure 4.3. It can be seen that the „BIOS" test results the greatest ratio of failed elements, whilst the „Power" test is the most reliable considering it outputs the least failures among all the steps.

To solve the scheduling problem, a mixed-integer non-linear mathematical solver is required. SCIP [69] [45] is currently one of fastest non-commercial solver for mixed integer programming and mixed integer nonlinear programming. It allows total control and access to detailed information about the solver. The optimisation steps of the given mathematical model were performed using the SCIP solver (Solving Constraint Integer Programs) on the NEOS server (https://neos-server.org/neos/), which is a free internet-based service for solving numerical optimisation problems.

The solution of the optimisation task Eqs. (4.13)-(4.22) is presented in the rest of the section using four special cases.

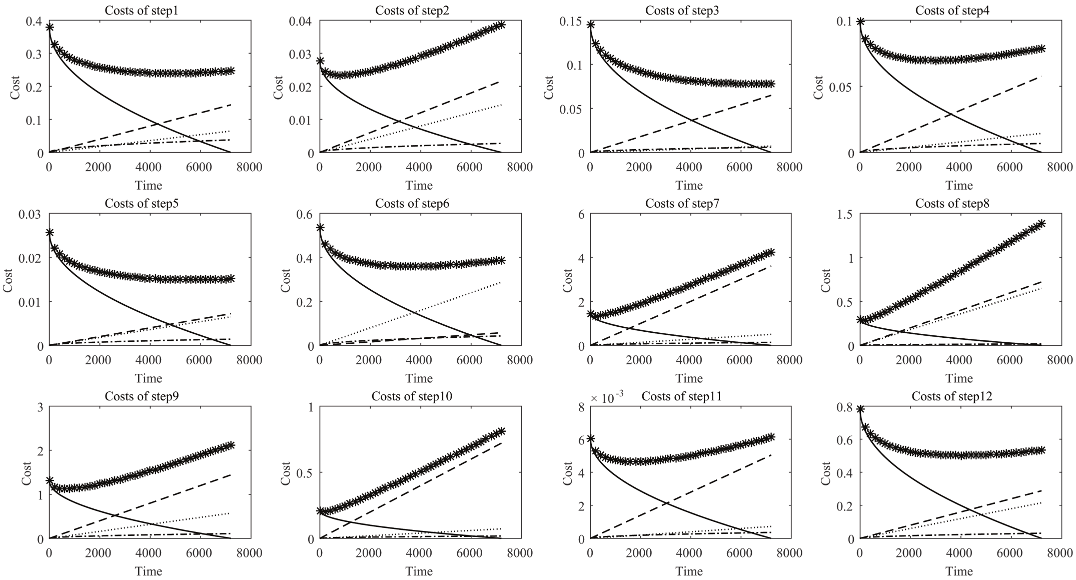

Figure 4.3: Cost functions of each step (line with $*$ marker: total cost function; dashed: fix cost function; dotted: proportional cost function; dash-dot: repair cost function; solid line: warranty cost function)

Case 1:

Let the vector representing the order of the test steps be $\pi = [1, 2, 3, \ldots, 11, 12]$, and $t_1 = t_2 = \ldots = t_{11} = t_{12} = 3600$ (the middle of the $[0, 7200]$ interval).

In this case, we only need to evaluate the objective function (Eq. (4.22)), since the value of each variable is determined. Table 4.4 shows the results of the evaluation: at the end of the testing process, 97.15% of the input items passed the tests with total cost \$78,181. The most expensive test step is #7: VGA checking, where 138 failed items were found. The high cost of this test step is due to its survival function with „relatively” low value of λ_7 and the high-cost parameters.

Case 2 allows modifying the testing time of each test step, while the testing order is still fixed.

Table 4.4: The result of Case 1 with $N_{in} = 20000$

Test step	Position in the test order	Duration of the test step (t_i)	Total cost of the test step	Nr. of elements leaving the test step
#1	1	3,600	4,908	19,946
#2	2	3,600	344	19,942
#3	3	3,600	1,648	19,930
#4	4	3,600	1,392	19,918
#5	5	3,600	313	19,916
#6	6	3,600	8,003	19,829
#7	7	3,600	24,503	19,691
#8	8	3,600	8,307	19,661
#9	9	3,600	16,040	19,524
#10	10	3,600	3,549	19,502
#11	11	3,600	82	19,501
#12	12	3,600	9,092	19,430
Total cost:			78,181	

Case 2:

Let the order vector of the test steps be
$\pi = [1, 2, \ldots, 11, 12]$, and $t_1 = min_{0<t\leq 7200}\, C^1(t)$, $t_2 = min_{0<t\leq 7200}\, C^2(t)$, $t_3 = min_{0<t\leq 7200}\, C^3(t)$, $\ldots$, $t_{11} = min_{0<t\leq 7200}\, C^{11}(t)$, $t_{12} = min_{0<t\leq 7200}\, C^{12}(t)$.

Although the value of the objective function is not independent of the number of tested items, the minimums of the $C^i(t)$ functions can be useful for estimating the test durations. As shown in Table 4.5, the minimum time of each total cost function was determined, and the objective function was evaluated. As a result of this modification, the total cost of the testing process decreased by 7.5% even though the test sequence was still fixed.

Table 4.5: The result of Case 2

Test step	Position in the test order	Duration of the test step ($t_i = min_{0<t\leq 7200} C^i(t)$)	Total cost of the test step	Nr. of elements leaving the test step
#1	1	4,500	4,878	19,940
#2	2	5,388	336	19,935
#3	3	6,708	1,552	19,918
#4	4	2,971	1,385	19,908
#5	5	5,111	308	19,905
#6	6	2,340	7,837	19,835
#7	7	1,493	22,709	19,746
#8	8	332	5,228	19,737
#9	9	5,029	15,868	19,575
#10	10	1,409	3,261	19,561
#11	11	3,134	82	19,560
#12	12	5,483	8,889	19,472
Total cost:			72,334	

Case 3:

Contrary to the cases before, the order of the test steps is optimized in Case 3, but test durations are still fixed; the time minimums are chosen as follows: $t_1 = min_{0<t\leq 7200} C^1(t), t_2 = min_{0<t\leq 7200} C^2(t), \ldots, t_{11} = min_{0<t\leq 7200} C^{11}(t)$, $t_{12} = min_{0<t\leq 7200} C^{12}(t)$. By fixing the continuous variables in connection with the durations of a step, we get a linear objective function. The solution got is a cost of \$72.091, which is a further 3.35% improvement compared to the result of Case 2. Solving the problem with SCIP optimiser the suggested optimal sequence is given as $\pi = [11, 2, 1, 12, 5, 9, 3, 4, 6, 10, 8, 7]$ where the most expensive step is #7 and it is the last in the sequence.

Table 4.6: The result of Case 3

Test step	Position in the test order	Duration of the test step ($t_i = min_{0<t\leq 7200}\, C^i(t)$)	Total cost of the test step	Nr. of elements leaving the test step
#1	3	4,500	4,877	19,934
#2	2	5,388	337	19,994
#3	7	6,708	1,532	19,663
#4	8	2,971	1,367	19,652
#5	5	5,111	307	19,841
#6	9	2,340	7,738	19,583
#7	12	1,493	22,394	19,472
#8	11	332	5,181	19,560
#9	6	5,029	15,953	19,679
#10	10	1,409	3,263	19,569
#11	1	3,134	84	19,999
#12	4	5,483	9,059	19,844
Total cost:			72,091	

Case 4:

In the last scenario the order and the duration of the test steps are also optimized, where $0 < t_i \leq 7200\,(i = 1, \ldots, 12)$.

When also time and test step order optimisation is done, the optimal sequence is $\pi = [2, 11, 5, 4, 9, 3, 12, 1, 10, 6, 8, 7]$, which differs from the solution got in Case 3. The optimal cost obtained is \$72.062, which is a 0.04% and a 0.376% improvement compared to Case 3 and Case 2. When solving Case 4 it is worth to start the optimisation method from the initial feasible solution of Case 3.

Notice that, the optimal duration of the test step i is close to the minimum of $C^i(t)$ function, thus the results of special cases of the optimisation problem can

Table 4.7: The result of Case 4

Test step	Position in the test order	Optimal duration of the test step (t_i)	Total cost of the test step	Nr. of elements leaving the test step
#1	2	4,840	4881	19,932
#2	1	5,869	338	19,995
#3	3	6,980	1,552	19,916
#4	9	3,057	1,362	19,574
#5	8	5,219	303	19,585
#6	7	2,437	7,741	19,588
#7	11	1,498	22,394	19,472
#8	12	331	5,155	19,463
#9	4	5,236	16,016	19,749
#10	10	1,442	3,261	19,560
#11	6	3,268	82	19,658
#12	5	5,637	8,976	19,659
Total cost:			72,062	

be an appropriate starting point to the Eqs. (4.13) -(4.22). In lack of the results of special cases, the optimisation method works in a larger searching space, which results in longer running time of the algorithm. However, the results show that the optimisation can improve both the reliability of quality control process and cost efficiency factors.

The realistic case study taken from computer manufacturing demonstrated 0.1-5% cost reduction thanks to the optimal re-ordering of the test sequence. The methodology can be extended to a wide range of risk-based process optimisation tasks, like scheduling maintenance, medical diagnostic, or quality checking tasks.

4.6 Major results and related publication

In modular, replaceable unit based production systems, complex testing processes should be synthesised from independent test steps to ensure the desired product quality with minimum average cost. To reduce the time and cost spent on testing whilst minimising the risk of not detecting faults, a survival analysis-based cost models for elementary tests to combine the time-dependent outcome of the tests and costs related to the operation of the testing system were proposed. A mixed integer non-linear programming model to formalize how the total cost of testing depends on the sequence and parameters of the elementary test steps was introduced. To provide an efficient formalization of the scheduling problem and avoid difficulties due to the evaluation of an objective function during the relaxation of the integer variables, the MINLP was formulated as a process network synthesis problem. A P-graph, as stated above giving the structure and the mathematical model of a considered test process, consisting of three levels was constructed involving all the feasible test sequences as its substructures, while the cost parameters and functions efficiently represented the objectives. The mathematical model was automatically generated from the structural representation, and the SCIP (Solving Constraint Integer Programs) solver was applied to provide the optimal solution for the test-sequence problem. A realistic case study taken from the computer manufacturing industry demonstrated a cost reduction of 0.1-5% thanks to the optimal re-ordering of the test sequence. This methodology can be extended to a wide range of risk-based process optimisation tasks, like scheduling maintenance, medical diagnostics, or quality control.

Related publication

János Baumgartner, Zoltán Süle, Botond Bertók, János Abonyi: Test-sequence optimisation by survival analysis, Central European Journal of Operations Research, 27(2), 357-375, 2019. IF=2.23, Rank: **Q2**

4.7 Thesis 3

I have developed a survival analysis-based cost model to formalize how the total cost of testing depends on the sequence and parameters of the elementary test steps. I have constructed a P-graph consisting of three levels involving all the feasible test sequences as its substructures, while the cost parameters and functions efficiently represented the objectives.

- I formalized the test-sequence optimisation problem as a process network synthesis problem which gives the optimal order and duration of the tests have to be determined, where the constraints should guarantee the right ordering of test steps.
- For a P-graph-based description of the order of the test steps, I have extended the mathematical model where three levels of operating units and survival function-based malfunctions illustrate the feasible test sequences.
- To demonstrate the applicability of the proposed methodology, I have introduced an illustrative example related to the sequence optimisation problem, where survival functions represented by Weibull distribution describe the ratio of items that successfully passed and failed the test steps. The method has resulted in a cost reduction of 0.1–5% through the optimal rescheduling of test series.

Chapter 5

Summary

In my dissertation, I have investigated optimisation tasks related to Industry 4.0, which, in my opinion, can further increase the efficiency and reliability of manufacturing processes. Today's economic environment increasingly requires a reduction in the use of human resources and a continuous increase in the efficiency of manufacturing processes. Modern IT tools, large data sets generated in real-time and their rapid processing allow us to have up-to-date information on the internal processes of the industrial environment and to intervene appropriately when negative trends are detected in the achievement of short or long-term objectives. All this intervention and planning can be effectively supported by optimisation, which provides a wide range of algorithms and methods to achieve efficiency gains.

I have provided new methods and scientific results focusing on Industry 4.0 and optimisation in my work. In an industrial environment, ensuring the continuous operation of equipment is of essential importance. This topic raises many issues in reliability theory. In the industrial environment, the success or failure of the processes under investigation is often supported by success tree and fault tree descriptions and related computational methods. The drawback of these methods is that, in most cases, they are much less effective in supporting the answering of optimisation questions. Therefore, it is helpful to develop a framework algorithm that can automatically determine the reliability of a process or system topology using P-graphs and algorithmic solutions while allowing for several other constraints. In this context, I have investigated the reliability redundancy allocation problem and adapted the methodology for calculating reliability to the P-graph methodology.

For this purpose, I determine the path set and cut set elements by my developed algorithm for P-graph problems, which allows me to study both reliability and cost-based optimisation issues. The use of redundancy is a trivial way to increase the reliability of systems, but determining the right degree of redundancy can depend on many aspects. In my thesis, I have also shown that the time-dependent characteristic of reliability should be given maximum consideration in optimisation, and I have defined a set of measures that can be used to classify the subsystems of a system related to reliability. All these results are illustrated by real case studies from the literature and their computational results.

Effective planning of quality assurance tasks can further increase the effectiveness of a company. For example, efforts should be made to identify most potential customer complaints before delivery. For this reason, the testing steps for these manufactured products define a scheduling task. My aim in this context was to plan the sequence and timing of the tests that optimally determine quality assurance, thus ensuring a cost-effective operation. To study this task, I adopt a novel approach by describing the variation of the failure over time using Survival functions in the P-graph environment. This solution leads us to a non-linear optimisation problem analysed and solved in several fundamental requirements. The mathematical model I have developed is based on the testing practice of a real Hungarian manufacturing company, where my developed procedure was able to improve the time to identify defective products to a large extent. The results have been presented in the last part of my dissertation.

I have summarised the results of my research in this dissertation and three thesis points. The novel solutions developed have been published in international journal papers and conference proceedings and cited by the international research community in 26 scientific publications.

Chapter 6

Summary in Hungarian (Összefoglalás)

Disszertációmban olyan Ipar 4.0-hoz kötődő optimalizálási feladatok vizsgálatával foglalkozom, amelyek tovább növelhetik a gyártási folyamatok hatékonyságát és megbízhatóságát. Napjaink gazdasági környezete egyre inkább megköveteli az emberi erőforrások felhasználásának mérséklését, valamint a gyártási folyamatok hatékonyságának folyamatos növelését. A korszerű IT eszközök, a valós időben előálló nagy méretű adathalmazok és gyors feldolgozásuk lehetővé teszik azt, hogy naprakész információval rendelkezzünk az ipari környezet belső folyamatairól, és megfelelő módon avatkozzunk be akkor, amikor negatív tendenciát észlelünk a rövid, vagy éppen hosszútávú célok elérése kapcsán. Mindezen beavatkozást és tervezést hatékonyan tudja támogatni az optimalizálás, amely algoritmusok és módszerek széles tárházát biztosítja a hatékonyságnövelés elérése érdekében. Munkámban ilyen, Ipar 4.0 és optimalizálás fókuszú tématerületek kapcsán adok meg új módszereket és tudományos eredményeket. Ipari környezetben kiemelten fontosak a munkamenet-folytonosságot garantáló berendezések folyamatos működéseinek biztosítása. E témakör a megbízhatóságelmélet számos kérdését veti fel. Ipari környezetben a vizsgált folyamatok sikeres vagy sikertelen lefutását sok esetben a sikerfa és hibafa leírásokkal, valamint az ezekhez kapcsolódó számítási módszerekkel támogatják. A módszerek hátránya legtöbb esetben az, hogy az optimalizálási kérdések megválaszolását sokkal kevésbé támogatják hatékonyan, ezért indokolt egy olyan keretalgoritmus kidolgozása, amely képes automatikusan, algoritmikus eszközök felhasználásával meghatározni egy gráfok segítségével felírt folyamat vagy rendszertopológia megbízhatóságát, miközben számos egyéb korlátozó feltétel figyelembevételére is lehetőség adódhat. E témakörhöz kapcsolódóan a rendszermegbízhatóság növelésének módjaként a redundancia szerepét vizsgálom meg, és adaptálom a megbízhatóság számítására alkalmas metodikákat P-gráf környezetbe. Ehhez az ún. vágat halmazok elemeit határozom meg saját algoritmussal egy P-gráffal megadott feladatra, amely lehetőséget ad mind a megbízhatóság, mind a költség alapú optimalizálási kérdések vizsgálatára. A redundancia alkalmazásával triviális módon növelhető a rendszerek megbízhatósága, viszont a megfelelő mértékű redundancia meghatározása sok szemponttól függhet. Dolgozatomban rámutatok arra, hogy a megbízhatóság időfüggő jellemzőjét érdemes az optimalizálás során maximálisan figyelembe venni, és olyan mértékeket is definiálok, amelyek egy rendszer alrendszereit képesek megbízhatóság szempontjából minősíteni. Mindezen eredményeimet valós, irodalomból vett esettanulmányokkal és azok számítási eredményeivel támasztom alá.

A minőségbiztosítási feladatok hatékony tervezése tovább növelheti egy vállalat eredményességét. A vevői kiszállítást megelőzően arra kell törekednünk, hogy a legtöbb lehetséges vevői kifogást már a kiszállítás előtt azonosítsuk. Éppen ezért a gyártott termékek kapcsán azok tesztelési lépései egy ütemezési feladatot határoznak meg. Célom ennek kapcsán az, hogy a minőségbiztosítást meghatározó tesztelések sorrendjét és idejét optimális módon tervezzem meg, ezáltal biztosítva a költséghatékony működést. A feladat vizsgálatára egy újszerű megközelítést alkalmazok, hiszen P-gráf környezetben túlélési függvények segítségével írom le a meghibásodás időbeli változását. Mindez egy nemlineáris optimalizálási feladatot eredményez, amelyet több, valós követelmény szempontjából elemzek, és oldok meg. A kidolgozott matematikai modell alapját egy valós, Magyarországon működő gyártóvállalat tesztelési gyakorlata adta, amely során a kidolgozott eljárás segítségével nagy mértékben csökkenthető a hibás termékek azonosítására fordított idő.

Kutatómunkám eredményeit disszertációmban és három tézispontban foglalom össze. A kidolgozott újszerű megoldásokat nemzetközi folyóiratcikkekben és konferenciakötetekben publikáltam, melyekre a nemzetközi kutatói közösség már számos alkalommal hivatkozott.

Chapter 7

Summary in German (Zusammenfassung)

In meiner Dissertation untersuche ich Optimierungsaufgaben im Zusammenhang mit Industrie 4.0, die meiner Meinung nach die Effizienz und Zuverlässigkeit von Fertigungsprozessen weiter steigern können. Das heutige wirtschaftliche Umfeld erfordert zunehmend eine Reduzierung des Personaleinsatzes und eine kontinuierliche Steigerung der Effizienz von Fertigungsprozessen. Moderne IT-Werkzeuge, groSSe, in Echtzeit generierte Datensätze und deren schnelle Verarbeitung ermöglichen es uns, über aktuelle Informationen über die internen Prozesse des industriellen Umfelds zu verfügen und angemessen einzugreifen, wenn negative Trends bei der Erreichung kurz- oder langfristiger Ziele festgestellt werden. All diese Eingriffe und Planungen können durch diese Optimierungen wirksam unterstützt werden, da sie eine breite Palette von Algorithmen und Methoden zur Erzielung von Effizienzgewinnen bietet.

In meiner Arbeit stelle ich neue Methoden und wissenschaftliche Ergebnisse zum Thema Industrie 4.0 und Optimierungen vor. In einem industriellen Umfeld ist die Sicherstellung des kontinuierlichen Betriebs von Anlagen von wesentlicher Bedeutung. Dieses Thema wirft viele Fragen der Zuverlässigkeitstheorie auf. Im industriellen Umfeld wird der Erfolg oder Misserfolg der zu untersuchenden Prozesse oft durch Erfolgs- und Fehlerbaumbeschreibungen und damit verbundene Berechnungsmethoden unterstützt. Der Nachteil dieser Methoden ist, dass sie in den meisten Fällen bei der Beantwortung von Optimierungsfragen weit weniger effektiv sind. Daher ist es hilfreich, einen Rahmenalgorithmus zu entwickeln, der die Zuverlässigkeit einer Prozess- oder Systemtopologie mit Hilfe von P-Graphen und algorithmischen Lösungen automatisch bestimmen kann und dabei verschiedene andere Einschränkungen berücksichtigt. In diesem Zusammenhang untersuche ich das Zuverlässigkeits-Redundanz-Zuordnungsproblem und passe die Methodik zur Berechnung der Zuverlässigkeit an die P-Graphen-Methodik an. Zu diesem Zweck bestimme ich die Pfad- und Schnittmengenelemente mit dem von mir entwickelten Algorithmus für P-Graphen-Probleme, der es mir ermöglicht, sowohl Zuverlässigkeits- als auch kostenbasierte Optimierungsfragen zu untersuchen. Die Verwendung von Redundanz ist ein trivialer Weg, um die Zuverlässigkeit von Systemen zu erhöhen, aber die Bestimmung des richtigen Redundanzgrades kann von vielen Aspekten abhängen. In meiner Arbeit zeige ich, dass die zeitabhängige Eigenschaft der Zuverlässigkeit bei der Optimierung maximal berücksichtigt werden sollte und ich eine Reihe von MaSSnahmen definiere, die zur Klassifizierung der Teilsysteme eines Systems in Bezug auf die Zuverlässigkeit verwendet werden können. Alle diese Ergebnisse werden durch reale Fallstudien aus der Literatur und deren Berechnungsergebnisse illustriert.

Eine effektive Planung von Qualitätssicherungsaufgaben kann die Effektivität eines Unternehmens weiter erhöhen. So sollte beispielsweise versucht werden, die meisten potenziellen Kundenreklamationen vor der Auslieferung zu erkennen. Aus diesem Grund stellen die Prüfschritte für diese hergestellten Produkte eine Planungsaufgabe dar. In diesem Zusammenhang ziele ich darauf ab, die Reihenfolge und den Zeitpunkt der Tests so zu planen, das die Qualitätssicherung optimal bestimmt wird und somit ein kosteneffizienter Betrieb gewährleistet werden kann. Um diese Aufgabe zu untersuchen, wähle ich einen neuartigen Ansatz, indem ich die Variation des Fehlers über die Zeit mit Hilfe von Survival-Funktionen in der P-Graph-Umgebung beschreibe. Diese Lösung führt mich zu einem nichtlinearen Optimierungsproblem, das in mehreren grundlegenden Anforderungen analysiert und gelöst wird. Das von mir entwickelte mathematische Modell basiert auf der Prüfpraxis eines realen ungarischen Fertigungsunternehmens, in dem das von mir entwickelte Verfahren die Zeit bis zur Identifizierung fehlerhafter Produkte in hohem MaSSe verbessern kann.

Ich fasse die Ergebnisse meiner Forschung in meiner Dissertation in drei Thesen zusammen. Die entwickelten neuartigen Lösungen wurden in internationalen Zeitschriften und Konferenzberichten veröffentlicht und von der internationalen Forschungsgemeinschaft in mehreren wissenschaftlichen Publikationen zitiert.

Chapter 8

New Scientific Results

Thesis 1

I have adapted the fault- and success tree-based methodology of reliability calculation to the P-graph framework. The developed approach allows the algorithmic reliability-based analysis of processes given by P-graph descriptions.

- I have developed an algorithm for generating minimal path sets of P-graph processes, which allows the calculation of process reliability.
- I have built a P-graph-based optimisation model to solve the reliability-redundancy allocation problem. The evaluation of the objective function in the implemented model is calculated by computing the minimal path sets of P-graphs.
- I have validated the results of the P-graph-based mathematical model by solving a real case study of the literature. Based on the fault- and success tree of a real reaction system, a polynomial risk model has been developed, and reliability optimisation, as well as, computation of the number of the redundant elements has been performed.

Thesis 2

I have generalized the methodology of P-graph-based reliability calculation to the time-dependent case and k-out-of-n ($k$oo$n$) configurations. The developed model can be easily applied to calculate risks related to the malfunction of the subsystems and determine the risk-based importance of the process units.

- I have defined cut set-based metrics for the identification of the critical elements in a system.
- I have developed a multi-objective formulation of the $k$oo$n$ redundancy allocation problem, where time dependent reliability of a given system, the degree of the redundancy, and the related costs can be optimized.

Thesis 3

I have developed a survival analysis-based cost model to formalize how the total cost of testing depends on the sequence and parameters of the elementary test steps. I have constructed a P-graph consisting of three levels involving all the feasible test sequences as its substructures, while the cost parameters and functions efficiently represented the objectives.

- I formalized the test-sequence optimisation problem as a process network synthesis problem which gives the optimal order and duration of the tests have to be determined, where the constraints should guarantee the right ordering of test steps.
- For a P-graph-based description of the order of the test steps, I have extended the mathematical model where three levels of operating units and survival function-based malfunctions illustrate the feasible test sequences.
- To demonstrate the applicability of the proposed methodology, I have introduced an illustrative example related to the sequence optimisation problem, where survival functions represented by Weibull distribution describe the ratio of items that successfully passed and failed the test steps. The method has resulted in a cost reduction of 0.1–5% through the optimal rescheduling of test series.

Chapter 9

New Scientific Results in Hungarian

Új tudományos eredményeim:

1. Tézis

Adaptáltam a hiba- és sikerfa alapú hagyományos megbízhatóságszámítási eljárásokat a folyamathálózat-szintézis feladatok optimalizálására alkalmas P-gráf keretrendszerbe. A kidolgozott megközelítés algoritmikusan lehetővé teszi a P-gráf leírásokkal definiált folyamatok megbízhatóság alapú elemzését és optimalizálását.

- Kidolgoztam egy algoritmust P-gráffal leírt folyamathálózat-szintézis feladatok minimális vágat halmazainak generálására, amely lehetővé teszi a tekintett feladat megbízhatóság alapú vizsgálatát és annak optimalizálását.

- Matematikai modellt adtam meg, amely P-gráf alapú optimalizációs feladatok esetén redundancia alkalmazásával írja le a rendszermegbízhatóság növelését célzó követelményeket, ahol a modell célfüggvénye az általam kidolgozott minimális vágatokat előállító halmazok eredményire támaszkodva minősíti a lehetséges megoldásokat.

- A P-gráf alapú optimalizációs modell eredményeit valós szakirodalmi esettanulmány megoldásával validáltam. Egy valós kémiai folyamatok működését bemutató rendszer hiba- és sikerfája alapján polinomokkal leírt kockázati modellt dolgoztam ki, és elvégeztem a megbízhatóság optimalizálását, valamint a redundáns elemek számának meghatározását.

2. Tézis

A P-gráf alapú megbízhatóságszámítás módszertanát általánosítottam időfüggő esetre, valamint a k-out-of-n ($koon$) konfigurációkra. A kidolgozott modell könnyen alkalmazható a különféle rendszerek hibás működésével kapcsolatos kockázatok számszerűsítésére, és azok kockázatalapú fontosságának meghatározására

- A P-gráf leírással adott műszaki rendszerek kritikus elemeinek azonosítására és számszerű jellemzésére metrikákat vezettem be, amelyek alapját a minimális vágat halmazok elemei adják.
- Többcélú optimalizálást biztosító matematikai modellt dolgoztam ki a k-out-of-n típusú redundancia meghatározására, ahol a tekintett rendszer időfüggő megbízhatósága, a redundancia mértéke, és a kapcsolódó költségek együttesen vehetők figyelembe az optimalizálás során.

3. Tézis

Kidolgoztam egy túlélés elemzésen alapuló költségmodellt elemi tesztelési lépéseket tartalmazó minőségbiztosítási folyamatok optimálására, ahol a költségelemek csak az elemi vizsgálati lépések sorrendjétől és paramétereitől függenek. A kidolgozott P-gráf alapú modell minden lehetséges tesztelési sorozatot tartalmaz, a bemenetként megadott költségparamétereket és költségfüggvényeket adaptáltam a folyamathálózat-szintézis alapú reprezentációhoz.

- A tesztsorozat-optimalizálási problémát folyamathálózat-szintézis feladatként formalizáltam, amely megadja a tesztek költségoptimális sorrendjét és időtartamát, valamint a modellben alkalmazott megkötések garantálják a tesztlépések helyes permutációját.
- A tesztlépések optimális jellemzőinek és sorrendjének felírásához egy három szintű P-gráf reprezentációt dolgoztam ki, ahol a műveleti egységekre felírt korlátozások, és a túlélési függvény alapú meghibásodási arányok modellbe illesztése biztosítja az optimális tesztsorozatokat.

- A javasolt módszertan gyakorlati hasznosítását egy valós példán szemléltettem, ahol Weibull-eloszlással írtam le azon túlélési függvényeket, amelyek a tesztlépéseken sikeresen átment és sikertelenül megbukott elemek arányát írják le az idő függvényében. A módszer a költségek 0.1-5%-os csökkentését eredményezte a tesztsorozatok optimális újraütemezése által.

Chapter 10

Publications

Scientific journal articles related to the thesis

- Zoltán Süle, János Baumgartner, János Abonyi, Reliability - Redundancy Allocation in Process Graphs, Chemical Engineering Transactions, 70: 991-996. (2018). Rank: **Q3**
- Zoltán Süle, János Baumgartner, Gyula Dörg, János Abonyi, P-Graph-Based Multi-Objective Risk Analysis and Redundancy Allocation in Safety-Critical Energy Systems, Energy, 179: 989-1003. (2019). Rank: **D1**, IF = 6.947
- János Baumgartner, Zoltán Süle, Botond Bertók, János Abonyi, Test-Sequence Optimisation by Survival Analysis, Central European Journal of Operations Research, 27 (2): 357-375. (2019). Rank: **Q2**, IF=2.23

International conference abstracts and papers

- János Baumgartner, Risk-focused Optimisation of Engineering Systems by Process Network Synthesys, Abstract book of the 15th International Conference on Advanced Computational and Experimenting, 99. (2022).
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Further journal and conference papers:

- Zoltán Süle, János Baumgartner, Dániel Leitold, Tibor Dulai, Ákos Orosz, Ágnes Vathy-Fogarassy, Software Framework and Graph-based Methodology for Optimal Patient Appointment Planning, Procedia Computer Science, (accepted on 20 July 2022). (2022).
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