



**SZENT ISTVÁN UNIVERSITY**  
**PHD SCHOOL OF MANAGEMENT AND BUSINESS ADMINISTRATION**

The Interpretation of Sustainability Criteria using Game Theory Models

Theses of PhD Dissertation

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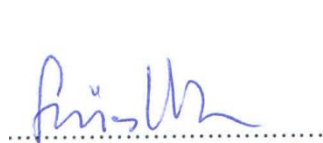
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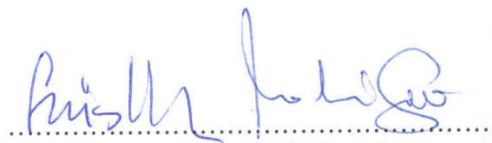
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# 1. Antecedent and Goals

It is very difficult to calculate in advance the positive and negative long-term impacts of an investment, or a development venture. A serious global problem arises from the fact that numerous environmental-protection oriented private and government ventures are implemented in an incorrect manner significantly impair the conditions of both the environment and the economy (market). There is a high number of innovative energy related investments, waste and water management projects, etc. in Europe, which cause more harm to the society than ever imagined. In Hungary, the state funding – be it direct or indirect – of such enviro-protection oriented ventures amounts to thousands of billions of HUF every year, and the improper use of these funds can set an entire economical sector back for decades in its development. Therefore for these funds, proper mapping, channeling in a manner conforming with market mechanisms, and sufficient re-structuring (as fast as possible) can not only liberate enormous resources for the society and the domestic economy, but can also contribute to the improvement of the current state of the environment, the labor market, and welfare indicators. For these aforementioned development strategies and investment programs to be contestable, the need for a „construction mechanism” that can fundamentally outrule these adverse processes and false directions of development throughout the course of planning and actualisation arises.

In my paper, I'm trying to find solutions to these problems of planning and development methodology, based on both the Rubik Logic and modern Game Theory applications.

A typical form of environmentally/economically harmful funding is when economic policy doesn't have sufficient information on the environmental damages and negative environmental effects caused by the infrastructure to be developed (f.e. electric systems, or regional waste management facilities). Because of the low transparency of enviro-orientated developments, both in an economic, and in a market sense, if we compare them to „no governmental financing” sectors, we can observe far worse conditions when we aim for an economic development which keeps market equilibrium intact. Without this balance, supply or demand can typically be either too high or too low.

The main goal of this dissertation is to make enviro-protection and climate friendly investments sustainably plannable, under both economic, and political instability. Therefore, my goal is to provide a scientifically comprehensive model of mathematic relationships based on project development and management systems, which can handle the aforementioned problems with its new approach, regardless of national borders.

To realise the main goal of my research, I composed and defined the following targets:

- Through the process of literature review, the classic and modern style introduction of the relations of economical value and sustainability.
- The mathematic interpretation of sustainability factors, introduction of the sustainable economic equilibrium or corporate strategies in a game theory approach, interpretations of finding classic balance points in non-cooperative game theory solutions.

- Introduction of conflict alleviation and compromise search through Game Theory methods, and the evaluation of its adaptability in sustainable cooperative corporate strategies.
- The sustainability interpretation of the structure and solution of Rubik's Cube, and the analysis of relations between sustainability and the Rubik solution algorithms.

The market errors which presently have a detrimental impact on the economic environment require a capable external – meaning it's positioned outside the market's condition systems, but has an impact on them nonetheless – condition system, because it's imperative for the actualisation of sustainable enviro-investments. The creation of a set of rules through the summary of external effects, and the definement and tipisation of the correct external effects of developing sustainable economic structures (and keeping said set of rules defineable through mathematic functions) is a social requirement, because it can define the presently used criteria of sustainability for both market shareholders and political decision-makers.

If we go by what was said above, when I defined my goals, I primarily found the verification of the following hypotheses to be important:

- H1: The relationships of project attributes which have an impact on the viability and sustainability of enviro-orientated or climate friendly investments can be defined through models.
- H2: When searching for a sustainable equilibrium using game theory methods, it is possible to map functions of relations between the compared attributes .
- H3: Multivariate test functions may be used to select attribute groups which dominantly affect the successful implementation of the project.
- H4: The various sustainability logics can be synchronised with the 3×3×3 Rubik's Cube solution algorithms, and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning.

I assume that the „*Layer by layer*” Rubik's Cube solution method, that is, the row to row solution can be used to model the sustainability criteria, in other words, during project planning (e.g. investments in renewable energy) the conformance with sustainability criteria can be maintained with this method.

According to the hypothesis on the solution algorithms of the Rubik's Cube, the parts rotated next to each other, meaning the project attributes which have an impact on each other, have a relation system which can be defined in mathematic terms, therefore, their point of balance (f.e. Nash's) can also be determined by game theory models (finite games , zero sum games, oligopolistic games, etc.).

## 2. Source and Method

### 2.1. Source

Throughout my research, I thoroughly read though, categorised and critically evaluated literature sources of both electronic and printed formats, which include domestic and international literature. The theoretic of literature was conducted on the different interpretations of sustainability, the major Game Theory solutions used in economic strategy planning, and the Rubik solutions. For the theoretical base of the research, the software analysis of the cube's inherent attributes was followed by creation of the actual professional database. The primary data used for the SMART (Simple Multi Attribute Ranking Technic) 3D analysis was generated through the synthesis of the research results of Cleantech Incubation Europe (CIE).

### 2.2. Method

To analyse the software aimed at solving Rubik's Cube, I used a SWOT analysis during my research, and to evaluate the processes of the Rubik's Cube solution algorithms, I used Theoretic process evaluation. For the sustainability interpretations of low-carbon development processes, I used content analysis, for which I employed the aid of the European Union's Low-carbon 2050 strategic guide (<http://www.roadmap2050.eu/>). The main point of the content analysis was to „ask” the social product named „A practical guide to a prosperous, low carbon Eusope”, and other professional documents which went through social control during the data collection, for the sake of obtaining empirical data. During the research on Game Theory algorithms I was searching for, I used process of tolerance, meaning I was researching the admissible differences between the attributes of the cube, and the parametrisation of the Game Theory functions.

To define the criteria-system of the low-carbon project development concept, I used the Churchman-Ackoff procedure. I examined the estimated usefulness of the factors, for the sake of optimalsing the ones out of all the factor groups which are most important, and most useful to the project. To determine the relative usefulness of all the factors, I created „usefulness-functions”, which can properly represent either the equality, or the hierarchy of the factor groups.

#### 2.2.1. SWOT analysis

*The goal of the SWOT analysis on solution-searching:*

In case of the solution-searching softwares which were examined, the goal of the SWOT analysis is to determine if the functions of the softwares are applicable to the input and output system attributes of the low-carbon project evaluation model, and if they satisfy the user expectations. According to the data at hand, the object of the analysis should be: how, and how much is the low-carbon innovation and incubation system based on the „About low-carbon economy” (LCE Ltd. 2011), and „Hubconcepts – Global best practice for innovation ecosystems” (Launonen, 2011) professional guidelines satisfied by the chosen software.

*Reason for the choice of method:*

The SWOT analysis offers a good opportunity to create an overview comparison, which has no exact attributes defineable in easily comparable dimensions. In itself, the SWOT analysis has no meaning, however, if it's part of a complex analysis, it can sufficiently facilitate thought process.

### 2.2.2. Theoretic process evaluation

With the solution algorithms of the 3×3×3 Rubik's Cube, the sustainability theories can be synchronised, and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. I theoretically evaluated the various solution processes, and investment planning levels parallel to them by following the solution levels and stages of the cube. After these various level-evaluations, I made „low-carbon interpretation” summaries.

The structure of the process evaluation is as follows:

- defining sector or level,
- theoretic evaluation,
- evaluation of process and results (interpretations),
- summarising the evaluation of process and results.

To show the various states of the cube, and to attach an explanation to the low-carbon interpretations, I used the Online Ruwix Cube Solver program.

### 2.2.3. Data collection for multi-dimension „low-carbon” development processes with content analysis

The content analysis can be categorised in the research methods as a so-called „no intervention” type. The biggest benefit of using this type of examination is that the people doing the research are a proper distance away from the actualisation of the problem, therefore, they can do the data collection without any chance of intervention in the process. In this case, there is no problem with our data collection process affecting the answer. Content analysis is a kind of data collection where we conduct information gathering and analysis using the designated document. It is also a kind of social analysis method, which is used for the examination of human messaging (Kérdő, 2008). I analysed the program documentations through the social inspection mechanics, and debates in the Union, which were relevant to both the low-carbon development concepts, and the topic of sustainability. The point of the content analysis was to „ask” the social products named the „A practical guide to a prosperous, low-carbon Europe”, the „National Energy Strategy 2030”, and the „Hungary's renewable energy plan 2010-2020” (NCST2010-2020 for short) professional documents which went through social control, for the sake of obtaining empirical data.

During the completion of the content analysis, I found it important to „ask” these social products which are mostly goal orientations and documents – as the source of my empirical data – in a way that avoids contradictions, and to highlight the preference indicators of green energy, or climate-friendly investments as common decision factors.

#### *Reason for the choice of method:*

A primary form of data collection, where we try to find an answer with the dissolution of contradictions in the professional documentation, or continue the examination by re-defining the contradictions.

### 2.2.4. Evaluation of Game Theory algorithms by process of tolerance and applicability

The evaluation of process of tolerance in the sense of engineering means the allowed maximum differentiation from the determined sizes, quantities or qualities. In the case of the Game Theory algorithms, I researched the following: which method is the same as the solution process model of the Rubik's Cube in terms of its attributes, and in what scale does it differ from it, while staying representative. For the Game Theory algorithms I was searching for, I used process of tolerance, meaning I was researching the admissible differences between the attributes of the cube, and the parametrisation of the Game Theory functions (Ligeti, 2006). During the research, I took



the various Game Theory algorithms, and analysed the models that can be linked to the rotation algorithms (interpretations) in the process of model-making.

#### *2.2.5. Determining the criteria and cube attributes with the Churchman-Ackoff method*

It is important to weight the attributes which have an impact on the development processes, to choose the groups that give the defining conditions of the development of the project, and the actualisation of the investments. In the method made by Churchman and Ackoff, there are two almost indistinct processes, if we go by the number of attributes. The first one was made for one to seven attributes, while the second one for more than seven attributes. The method is based on consecutive comparisons, and is also usable to select a lower number of attributes, which in our case is four. With the help of this method, I designated the main attributes of the various development processes, which I can assign to Rubik's Cube's four analysis areas (Orange, Red, Blue, Green). I named them „main analysis agents (attribute groups)”, and then divided them further into sub-pieces (small cube attributes), according to their usefulness.

#### *2.2.6. Applicability analysis of „usefulness-functions” in multi-dimension evaluation (SMART)*

In order to optimise the most important and most useful attributes of the criteria systems (attribute groups) that have an impact on Rubik's Cube low-carbon development, we have to evaluate the estimated usefulness of the attributes. To determine the relative usefulness of all the factors, I created „usefulness-functions”, which can properly represent either the equality, or the hierarchy of the attribute groups. These „usefulness-functions” designate a single number to all the stages, to show the preferability of each stage. By combining the consequences of the actions with its probability, we get the estimated usefulness for each action (Russel-Norvin, 2003).

By defining the usefulness in the attribute groups, I was able to determine which attributes will be 1, 2, or 3 dimension cube attributes, in other words – which can be assigned to the mid cube, the outer cube, or the cornercube.

I also added rank values, or SMART values to the attribute groups, or various attributes within the groups, which I analysed with a three-dimensional comparison. The research conducted by the three-dimensional depictions determined which of the analysed attributes can be tagged with a „not allowed difference”. For these attributes, and their correction of equilibrium, we can use the typed Game Theory model.

### 3. RESULTS

#### 3.1. SWOT analysis

The SWOT analysis already depicted and explained in the methodology part before, has the goal of making it clear to me, if the functions of the chosen software are applicable to the input and output expectations of the low-carbon project evaluation model. The analysis was conducted in accordance to classic SWOT analysis rules, therefore, I won't show the details, only the charts with the results. However, for the sake of understandability, I will give a short explanation on the various software functions.

Softwares evaluated with SWOT analysis:

- ✓ RUWIX PROGRAM
- ✓ RUBIKSOLVE PROGRAM
- ✓ SOLUTION SEARCHING LBL SOFTWARE

The solution searching SWOT evaluation of the Ruwix program (Chart 1) in accordance to the input and output expectations of the low-carbon project evaluation model:

**Chart 1: Ruwix program SWOT chart**

	POSITIVE TRAITS	NEGATIVE TRAITS
INTERNAL TRAITS	<p><b>STRENGTHS</b>                      Exceptional graphics and visual details, some mention it as the world's most advanced solution software. Offers solutions not only to Rubik's Cube, but many other logical games.</p>	<p><b>WEAKNESSES</b>                      Presently not compatible, since it uses different, faster algorithms than the layer by layer solution, which aren't the best for low-carbon solutions.</p>
EXTERNAL TRAITS	<p><b>OPPORTUNITIES</b>                      Because of its strengths, and the applicability, it would be beneficial to develop low-carbon specifications as well.</p>	<p><b>THREATS</b>                      Since the program runs in an online format, it isn't possible to add special data to it. Even in case of a low-carbon specification, syncing the free software with the pay-to-use SMART add-on makes it difficult to use.</p>

Source: self-made

The solution searching SWOT evaluation of the Rubiksolve program (Chart 2) in accordance to the input and output expectations of the low-carbon project evaluation model:

**Chart 2: Rubiksolve program SWOT chart**

	POSITIVE TRAITS	NEGATIVE TRAITS
INTERNAL TRAITS	<p><b>STRENGTHS</b>                      Fast, constantly developed, can use layer by layer method</p>	<p><b>WEAKNESSES</b>                      2D, can't interpret layer by layer logic at the input, other user functions are missing.</p>
EXTERNAL TRAITS	<p><b>OPPORTUNITIES</b>                      Easy plugin options offer good compatibility with low-carbon usage.</p>	<p><b>THREATS</b>                      Since it focuses on fast solutions, not all details can be understood by the users.</p>

Source: self-made

**Chart 3: Rubik's Cube Solution Search program SWOT chart**

	<b>POSITIVE TRAITS</b>	<b>NEGATIVE TRAITS</b>
<b>INTERNAL TRAITS</b>	<p><b>STRENGTHS</b></p> <p>The steps of conceptual and practical solutions are the same</p> <p>The layer by layer solution is followed through in the program</p> <p>Uses obvious advancement and correction steps</p> <p>Because of the easy programming, it's also easy to develop</p> <p>Every algorithm is also defineable in the steps of the low-carbon project evaluation model as well.</p>	<p><b>WEAKNESSES</b></p> <p>The visual interface is not up-to-date</p> <p>Slightly slow processing</p> <p>Not available in online format</p> <p>As of now, it can only solve the 3×3×3 Rubik's Cube.</p>
<b>EXTERNAL TRAITS</b>	<p><b>OPPORTUNITIES</b></p> <p>Visual interface</p> <p>Easy to sync with the SMART evaluation software plugin</p> <p>The definition of low-carbon domain requires no additional development on the software</p> <p>Because of the easy programming, it may prove to be cheap to be a newcomer on the market.</p>	<p><b>THREATS</b></p> <p>Quite an old development</p> <p>The program may seem slow, because it can't be accelerated properly because of a set of certain configurations</p> <p>„Easy to copy”.</p>

Source: self-made

The introduced Ruwix Solver and Rubiksolve applications are both the further developed versions of Kociemba's *Cube Explorer*, which was the basis of most Rubik's Cube fans' software development work and ideas since 2005. After reviewing the different solution programs, we can say that there is an option to bring in technically any new algorithm, but of course, the goal of all the developers was to give the competitors a program that offers the solutions with the highest possible procession speed, and lowest number of combinations necessary.

The Rubik's Cube Solution Search program (Chart 3) completes the cube with the seven solution levels defined by *MOHÓ*'s search program. During the evaluation of the methodology manual, we made it clear that this one is able to get to the completed stage, meaning the one side – one color state from any starting stage with the layer by layer method. Also, the process may be stopped at any given stage. The number of rotations varies by the starting stage, but usually it takes more than 70 rotations to complete the cube. However, from an easier starting point, it can reduce to a mere 40-45 rotations.

Also, by analysing the SWOT evaluations, it can be said that the swift strenghts/weaknesses/opportunities/threats chart prefers the hungarian-developed Rubik's Cube Solution Search program, which was optimised for the layer by layer algorithms. This Java-based application proved to be best in its functionality for the low-carbon project evaluation model's input and output expectations, also noted by the structural trait that the software's „State Area” pack designates almost the same solution levels, that the hand-solved algorithms do. (The other evaluated softwares designate almost completely different levels.)

### 3.2. The principles and sustainability relations of the layer by layer solution method

The various sustainability logics can be synchronised with the 3×3×3 Rubik's Cube's solution algorithms, and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. I theoretically evaluated the various solution processes, and parallel investment planning levels following the solution levels and stages of the cube. After these various level-evaluations, I made „low-carbon interpretation” summaries. To show the various states of the cube, and to attach an explanation to the low-carbon interpretations, I used the Online Ruwix Cube Solver program.

In 1980, Ernő Rubik wrote that the cube seems to be alive, as it comes into life while you rotate it in your hands. Rubik's Cube has three rows and three columns, and this can also have a symbolic, or even mystical meaning. If we look at the attributes of the various blocks, the 3×3×3 cube's sides, it's almost immediately obvious that in case of each side, we have system elements, or specific small cubes (mid cubes, outer cubes, and cornercubes) which hide a specific meaning, and keep this meaning in them, regardless of where we rotate them in the system. According to Ernő Rubik, the number „three”, through its special meaning, is even able to model life itself. It's able to show the relationship of man and nature, the process of creation, care and destruction, and the relations of cooperation between our resource systems (Rubik, 1981).

We may think that the solution to the „mystical cube game” problem may properly portray the biggest question of one of today's hardest problems – the proper and effective use of energy. Nowadays, the entire energy consumption system seems like a huge puzzle, where we don't seem to be able to find the correct pieces. However, we suggest that the 3×3×3 Rubik's Cube's solution method may help us find the various pieces' relations, the relevant inclusion of system attributes in both a 2D and 3D interpretable manner, therefore, it may give correct pointers on interpreting the supply and demand sides of energy consumption.

One of the most widely known and most used method of solving Rubik's Cube is the „layer by layer” method, but we must also note that it's the basis for the more advanced methods like Fridrich, Corner first, etc. The gist of the method is to complete the cube during the solution process row by row. That means that at first, we form a color cross on the first row, then insert the correct corners, then comes the middle row, and finally, the lower middle cube goes into its place, followed by the lower cornercubes (Fogarassy et al., 2012).

Most amateurs use the layer by layer method, since this is the easiest to learn, and this is one of the few that has both a professionally based algorithm, and introduction guides. All other advanced solution methods began from this one. I introduced the process of solution according to the outline provided by the [www.rubikkocka.hu](http://www.rubikkocka.hu) official website. However, in the current document, I also included UNFCCC's basic development theories, namely „Low-Emission and low-carbon Development Strategies” (LEDS) – which has close ties to basic sustainability criteria – for the official solution method cited in this document.

We made the assumption that since the Rubik's Cube's number „three” offers indirect answers to many of our world's currently unsolved questions though it's mystical logic, it's correct to also assume that those who can complete the cube can think „Rubically” in general, or more specifically, about the questions of strategic planning and economic equilibrium search. In the next part of this document, you can find the methodical steps on solving the cube, which can be taken as a compilation theory during strategic development following the solution of the cube, usable for f.e. the advancement from fossilized to renewable energy support systems.

### 3.2.1. Process evaluation of *layer by layer* completion of the 3×3×3 Rubik's Cube

The layer by layer method is fundamentally a structured arrangement system, which defines cornerstones, stages to the process of completion (white cross, second row, yellow cross, etc.), where even though these stages can be achieved by different routes, or one might say that everyone does it to their own personal leisure, it is technically impossible to advance to the next stage without going through the various stages and phases.

**Chart 4: Process of model development, evaluation of results**

CUBE INTERPRETATIONS (number of rotation algorithm)	LEVEL OF MODEL DEVELOPMENT	/LOW-CARBON/ PROJECT ATTRIBUTE IN QUESTION	CORRELATION WITH GAME THEORY
NO1	INPUT	„White cross” – defining the starting criteria	A stage defineable by an n-person zero sum game of infinite kind.
NO2	INPUT	„White corner” – defining the sustainable development routes, equilibrium-search, non-cooperative optimum	According to functions on Nash-equilibrium, non-cooperative strategy, defineable by games of finite kind.
NO3	MID CUBE	„Second row” – anchoring of relation points, achieving equilibrium, arranging two-dimensional attributes, positioning fixpoint	Positioning outer cubes is possible with conflict alleviation methods. Fixpoint positioning is advised to be done with zero sum game.
NO4	MID CUBE	„Yellow cross” – indirect synchronising of input/output sides	Defineable by oligopolistic games of finite kind, or method of equal compromise.
NO5	OUTPUT	„Yellow corner” – interpretation of sustainability attributes during the arrangement of outputs	Defineable by three-person game of infinite kind, needs Nash-equilibrium.
NO6	OUTPUT	„Yellow side edge-switch” – strict synchronising of input/output sides	Defineable by zero sum game, conflict alleviation method, and cooperative strategy.
NO7	OUTPUT	„Corner switch” – the phase of setting the final balance, achieving equilibrium, finalising sustainability attributes	Oligopolistic games by functions based on either cooperative equilibrium strategy or Nash-equilibrium. Cooperative strategy.

Source: self-made.

In case of the sustainability principles and low-carbon development concepts, the abidement by the steps of development phase to phase has importance, because even though the circumstances and the makings may define different routes to equilibrium search, the arrangement logic must be the same, wherever we search for the equilibrium points – be it Hungary, or China, etc. I relied upon the methodical guideline of the [www.rubikkocka.hu](http://www.rubikkocka.hu) official website, and the solution plans of Singmaster (1980) during the defining of the row by row solution phases. However, because of the low-carbon methodology connections, the process which is demonstrated and interpreted in this document differs greatly from these guides. To illustrate the various stages and different solution levels of the cube, I used the Online Ruwix Cube Solver program.

### 3.3. 1D, 2D and 3D problem management method based on Rubik's Cube

Project planning and development is basically a process optimisation, which is based on the collective handling of different attributes, in a way that the examined segments are placed into the most harmonic constellation compared to each other. In case of a supposed „low-carbon optimisation protocol”, there is a need to create four different determination areas (attribute groups), which can be associated with the  $3 \times 3 \times 3$  cube's different colored sides. Two opposing sides (white and yellow) would be our project's input and output sides. The attribute groups which determine our optimisation can be the following in a demonstrational project: optimisation of strategic goals system (red side), analysis of market opportunities (green side), the area of actualisation and technological criteria system (blue side), monetary effects (orange side), the attributes summarizing input-side goals (white side), and last, but not least, the attributes summarizing output-side goals (yellow side).

One of the most important characteristics of the low-carbon optimisation concept (based on the software development experiences from India), is that the analysis of various projects on multiple levels is based on the analysis of the relevant interactions of various pieces, therefore, by avoiding the analysis of irrelevant interactions spares tremendous time and effort (low-carbon solution). The system connections assigned to the various sides of the cube (outer cube characteristics, and cornercube characteristics) makes the direct examination of various attributes irrelevant, meaning that not all system connections actually have to „communicate” with each other. The „communications” between these system elements can therefore be reached by simple border-area connections, or through transferred system connections.

In the next chart (Chart 5) I summarized the characteristics of the input and output sides of an actual project. The typed goal of project development in this case is the advancement from fossilized energy resources to renewable ones, or its combined systems. The four main agents which were assigned to the four colored sides were decided upon by professional evaluation, individual weighting and process of dominance analysis in charts 6 and 9. Of these four colors, red represents „criteria of laws and regulations for strategic program development”, green represents „examination of market opportunities”, blue represents „technological criteria system”, and orange represents „summarisation of monetary effects”. Also, on the various sides (main agents), I defined cube characteristics which represent two or three individual attributes through the connection system of the cubes themselves. The characteristics, which are unimportant and unrelated to the development and actualisation of the project per se, are linked to the mid cubes, of which there is one on each side, with one defining attribute – obviously though, even if there are fixed cubes on the input and output sides as well, they aren't associated with main agents. The attributes which include two or three different factors were linked to the outer and cornercubes. The defining of the attributes and their association to the cubes was done by their usefulness. The attributes which are determinable one way (pointlike) were named 1D (x), those which are determinable two ways were named 2D (x,y), and those which are determinable three ways were named 3D (x,y,z) attributes. Even though in our following project, the choice of definition of Rubik's Cube's attribute sides and its cubes was random, it's still advised to examine their usefulness with some kind of function-like connection, because we can define the importance of the attributes and the preference comparisons. Also, to raise the sufficiency of the model, we employed a new method to weight criteria and define dominance.

Following the completion of the process of dominance analysis and the use of the „usefulness-function”, both the attributes of the main agents and the small cube attributes fall into place.

**Chart 5: Meanings of input (white/W) and output (yellow/Y) sides of Rubik's Cube**

SIDE COLORS	<p align="center"><b>MEANING OF COLORS</b></p> <p align="center">1D – single-simension trait (x)                  2D – dual-dimension trait (x,y)                  3D – tri-dimension trait (x,y,z)</p>
<p align="center"><b>WHITE (F)</b></p>	<p><b>INPUT:</b>                      Defining input requirements, basic system of the product or provision defined along the matching of state regulations and market.</p> <p><b>IMAGING OF WHITE SIDE:</b></p> <p>MID WHITE (1D)                      ✓ energy rationalisation (W)</p> <p>OUTER WHITE (2D)                      ✓ strategic base-connection (WR),                      ✓ basic technological requirement (WB),                      ✓ financing expectation (WO),                      ✓ basic market positioning (WG),</p> <p>CORNER WHITE (3D),                      ✓ basic requirement of payoff (OGW),                      ✓ sufficiency to technological criteria and funding instruments (OBW),                      ✓ syncing basic goals with technological threats and innovation priorities (BRW),                      ✓ designating strategically synced market segment at basic criteria (RGW).</p>
<p align="center"><b>YELLOW (Y)</b></p>	<p><b>OUTPUT:</b>                      Pareto optimal product or provision system outlined by taking into consideration the maximum values of resource-usage opportunities.</p> <p><b>IMAGING OF YELLOW SIDE:</b></p> <p>MID YELLOW (1D)                      ✓ resource-optimised energy consumption/profitable production (Y)</p> <p>OUTER YELLOW (2D)                      ✓ strategic congruitisation of energy and CO2 scale (RY),                      ✓ optimising technological threat minimalisation (BY),                      ✓ tax and benefit criteria in the energy-production system (OY),                      ✓ monetary criteria of artificial and actual advancement to the market (GY),</p> <p>CORNER YELLOW (3D)                      ✓ structure compatible with strategic goals systems, where „shelf-life” is also guaranteed (RYG),                      ✓ planning option sufficient both in monetary and technological terms – meaning technological solution which guarantees positive cost-benefit rate (OYB),                      ✓ production and provision conditions sustainable on the market (GYO),                      ✓ long-term and legit option, where the chosen technological solution supports the strategic goals to the utmost level (BYR),                      (endorsement of sustainability criteria through development).</p> <p>(Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)</p>

Source: self-made.

**Chart 6: Meanings of attribute side Red(R) of Rubik's Cube**

SIDE COLORS	<p align="center"><b>MEANING OF COLORS</b>                      1D – single-dimension trait (x)                      2D – dual-dimension trait (x,y)                      3D – tri-dimension trait (x,y,z)</p>
<b>RED (R)</b>	<p><b>Criteria of laws and regulations for strategic program development:</b>                      Providing the defining information, synergies, cooperations for the planned profile on a corporate, local, sectoral, regional or union economic policy level.</p> <p>MID RED (1D)                      ✓ realisation of local/corporate strategy (R)</p> <p>OUTER RED (2D)                      ✓ following and matching of marketing strategy and economic policy priorities (RG),                      ✓ defining technological systems for cost-clear versions, matching the techno-parameters of financing priorities to the project (RB),                      ✓ strategic congruisation of energy and CO2 scale (RY),                      ✓ strategic base-connection (RW)</p> <p>CORNER RED (3D)                      ✓ syncing basic goals with technological threats and innovation priorities (BRW),                      ✓ designating strategically synced market segment at basic criteria (RGW),                      ✓ structure compatible with strategic goals systems, where „shelf-life” is also guaranteed (RYG),                      ✓ long-term and legit option, where the chosen technological solution supports the strategic goals to the utmost level (BYR)</p> <p>(Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)</p>

Source: self-made.

**Chart 7: Meanings of attribute side Green(G) of Rubik's Cube**

SIDE COLORS	<p align="center"><b>MEANING OF COLORS</b>                      1D – single-dimension trait (x)                      2D – dual-dimension trait (x,y)                      3D – tri-dimension trait (x,y,z)</p>
<b>GREEN (G)</b>	<p><b>Examination of market opportunities:</b>                      Evaluation of market opportunities and positions in artificial and actual market segments.</p> <p>MID GREEN (1D)                      ✓ plannable price in supply and demand equilibrium (G)</p> <p>OUTER GREEN (2D)                      ✓ following and matching of marketing strategy and economic policy priorities (RG),                      ✓ effects of market changes on the financing system, analysisation of foreign currency risk factors, and global effects (OG),                      ✓ monetary criteria of artificial and actual advancement to the market (GY),                      ✓ basic market positioning (WG).</p> <p>CORNER GREEN (3D)                      ✓ structure compatible with strategic goals systems, where „shelf-life” is also guaranteed (RYG),                      ✓ production and provision conditions sustainable on the market (GYO),                      ✓ basic requirement of payoff (OGW),                      ✓ designating strategically synced market segment at basic criteria (RGW).</p> <p>(Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)</p>

Source: self-made.



**Chart 8: Meanings of attribute side Blue(B) of Rubik's Cube**

SIDE COLORS	<p align="center"><b>MEANING OF COLORS</b></p> <p align="center">1D – single-simension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)</p>
<b>BLUE (B)</b>	<p><b>Technological criteria system:</b> Matching of market opportunities and technological solutions. Research of the techno-risks and opportunities is advised.</p> <p>MID BLUE (1D) ✓ technological usage that abides by BAT technological requirements (B)</p> <p>OUTER BLUE (2D) ✓ defining technological systems for cost-clear versions, matching the techno-parameters of financing priorities to the project (RB), ✓ basic technological requirement (WB), ✓ optimising technological threat minimalisation (BY), ✓ the most cost-efficient technological solution with both high quality and innovation-level (BO).</p> <p>CORNER BLUE (3D) ✓ sufficiency to technological criteria and funding instruments (OBW), ✓ planning option sufficient both in monetary and technological terms – meaning technological solution which guarantees positive cost-benefit rate (OYB), ✓ syncing basic goals with technological threats and innovation priorities (BRW), ✓ long-term and legit option, where the chosen technological solution supports the strategic goals to the utmost level (BYR).</p> <p>(Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)</p>

Source: self-made.

**Chart 9: Meanings of attribute side Orange(O) of Rubik's Cube**

SIDE COLORS	<p align="center"><b>MEANING OF COLORS</b></p> <p align="center">1D – single-simension trait (x) 2D – dual-dimension trait (x,y) 3D – tri-dimension trait (x,y,z)</p>
<b>ORANGE (O)</b>	<p><b>Summarisation of monetary effects:</b> Type of financing, relevance of government tools, tax, foreign currency risks, liquidity questions.</p> <p>MID ORANGE (1D) ✓ time for payoff, corporate value (O)</p> <p>OUTER ORANGE (2D) ✓ tax and benefit criteria in the energy-production system (OY), ✓ financing expectation (WO), ✓ the most cost-efficient technological solution with both high quality and innovation-level (BO), ✓ effects of market changes on the financing system, analysation of foreign currency risk factors, and global effects (OG)</p> <p>CORNER ORANGE (3D) ✓ planning option sufficient both in monetary and technological terms – meaning technological solution which guarantees positive cost-benefit rate (OYB), ✓ basic requirement of payoff (OGW), ✓ sufficiency to technological criteria and funding instruments (OBW), ✓ production and provision conditions sustainable on the market (GYO).</p> <p>(Definitions: D=dimension, W=white, Y=yellow, G=green, R=red, B=blue, O=orange)</p>

Source: self-made.

### **3.4. Process of project development with Rubik's Cube using Game Theory Method interpretations**

The low-carbon project planning and project development using Rubik's Cube is a specially constructed planning concept which – as of now – is a one of a kind concept that can interpret factors with an impact on processes in 3D. For „setting” the equilibrium point of the economical or resource-usage of input and output sides, and to describe the relation between them, I used game theory solutions which weren't used for this purpose during scientific research before.

Used before the process of modeling, the evaluation of process of tolerance in the sense of engineering means the allowed maximum differentiation from the determined sizes, quantities or qualities. In the case of game theory algorithms, I researched the following: which method is the same as the solution process model of Rubik's Cube in terms of its attributes, and in what scale does it differ from it, while staying representative. For the Game Theory algorithms I was searching for, I used process of tolerance, meaning I was researching the admissible differences between the attributes of the cube, and the parametrisation of the Game Theory functions.

During the complex modeling, I analysed the Game Theory models one by one, and through the process of modeling I assigned the relevant models to the various rotation algorithms (interpretations). I separated the attribute groups of the cube to three different aggregations, which are INPUT side attributes, MID CUBE side attributes, and OUTPUT side attributes. I used Game Theory methods to determine the points of equilibrium between the three attribute groups. The gist of this was that where the attribute elements were tagged with a „not allowed difference” by the SMART (Simple Multi Attribute Ranking Technic) analysis, I listed parameters which lead to the points of equilibrium (Nash equilibrium) through strategic models. Both the analyses and the modeling were conducted via a three-stage system, therefore I also conducted the Game Theory modeling of the entire process on three levels, meaning the matching of three different types of Game Theory models (or three different cost-functions).

#### *3.4.1. Algorithms of input-side imaging*

The project begins in this phase. We can find the answer to the following question: what do we have to keep in mind when starting a project! The incorrect rotation of the first layer, or row of cubes results in incorrect continuation, therefore, we can't approach the next layer.

We can easily explain this with a simple energy-transaction. If we change our initial energy-supply system in a way that the old one still has a life expectation of 20-40% of its estimated use duration, then we may end up with a considerable financial loss if we intervene. To avoid ending up in such a situation, we can use a Nash equilibrium to calculate the optimal intervention time.

#### GAME THEORY MODELING OF INPUT SIDE (LEVEL 1)

Enviro-orientated developments are fundamentally against the economic development priority system (f.e. the program for lowering the greenhouse gases, and the one for usage of fossilized energy sources contradict each other, since the former promotes minimalisation of energy consumption, while the latter promotes the increased emission of pollutants). When planning the first layer, this can be used in the process of project planning in terms of regulation policy and financing policy (Illustration 1). We also have the same situation concerning the the waterbase-defense and the rising requirements of favored water-dependant energy plants.

In case of various projects, for the sake of realising clear business regulations and sustainable business strategies, we have to include the criteria of non-cooperative competitors as well. In this situation, it's incredibly hard to find the Nash equilibrium, but imperative nevertheless, since the project can't be further developed in a controversy.

**Definition:**

The definition of the Nash equilibrium is as follows:

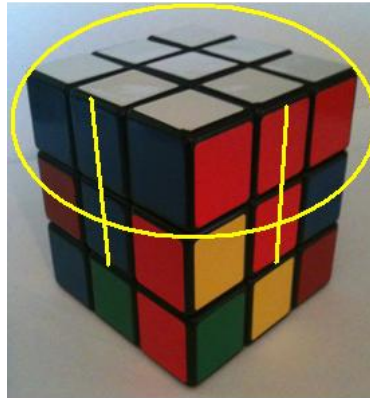
The equilibrium point or strategy of a  $J = (n, S, (\varphi_i)_{i=1}^n)$   $n$ -member game is a  $(x_1^*, \dots, x_n^*) \in S$  point (strategic  $n$ -tuple), which satisfies

$$\varphi_i(x_1^*, \dots, x_{i-1}^*, x_i^*, x_{i+1}^*, \dots) \geq \varphi_i(x_1^*, \dots, x_{i-1}^*, x_i, x_{i+1}^*, \dots)$$

for every  $i = 1, \dots, n$  player. Such an equilibrium is called a Nash equilibrium.

**Thesis:**

Following the completion of the first layer, only the connection with a Nash equilibrium can be further developed, meaning that we can only rotate the cube further from this position. The first layer always correlates with the second layer's mid cube, and can only be the same color.



**Illustration 1: Equilibrium point for the first row or layer (circled), where the mid cube is always the same color (illustrated by the lines).**

**Verification:**

Let  $x^* = (x_1^* \dots, x_n^*)$  be one point of equilibrium for the game. Then for any arbitrarily chosen  $y = (y_1 \dots, y_n) \in S$  :

$$\varphi_k(x_1^*, \dots, x_k^*, \dots, x_n^*) \geq \varphi_k(x_1^* \dots, y_k, \dots, x_n^*) \quad (k = 1, 2, 3 \dots, n),$$

from where through simple addition it's obvious that  $\varphi(x^*, x^*) \geq \varphi(x^*, y)$ . Based on this, a well-performing algorithm can be provided to define the points of equilibrium which have an impact on the planning, and to solve the fix-problems of the aggregations.

**Example:**

It's a critical point during the planning of biomass-based renewable energy production that the high amount of water consumed can have a detrimental effect on the project's profitability, and may become the criteria for use of the most effective technology. Therefore, the question, and the criteria is viewed as strictly technological in nature, and we try to match the strategy and Game Theory optimum with the cornercube which has 3D attributes (colours are red-green-white), where white means input, red means regulation criteria, and green means technological solutions, which we handle collectively (Illustration 2).

Luckily, solving water distribution problems plays a major role in Game Theory solutions, but we can usually reach the points of equilibrium that provide criteria for the outlines of an assured system usage only through defining many intricate function-connections, for which calculating mathematic correlations is quite difficult. Multi-purpose water usage, and the interests and cost-functions of those connected to it offer different optimums, which usually suppose a game of multi-player and nonlinear nature, and yet which is somehow still a non-cooperative game based on some kind of Nash equilibrium.

To define the problem – according to the low-carbon developments using Rubik’s Cube – I made a three player optimisation regarding water usage for the process of strategic planning using Rubik’s Cube, based on the guide by Szidarovszky and Molnár (2013).

Multi-purpose water usage as a decision-method task has been a problem for decades, and one with many solution options. In our case, we’re searching for one on a non-cooperative three-player (agricultural consumer /watering/, industrial consumer /cooling/, and household consumer /functional/) Nash equilibrium. The central element of the low-carbon strategy problem is how will the agricultural (biomass producer) water usage project developer decide, if the project has enough water out of the resources at hand.

SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATTRIBUTES
73	cornercube with more dominating attribute	basic criteria designation of matching set of strategies and technologies (WGR)
57	edge cube with less dominating attribute	strategy, regulation base connection (WG)
87	cornercube with more dominant attribute	abidement by technological criteria and funding instruments (WOG)

**Illustration 2: 3D attributes of „white-green-red” corner cubes (WGR), the technological solution that assures payoff (optimised for three-person water usage) (Dimensions from left to right: SMART value, Cube type and dominance, Main agent inherent attributes) Source: self-made.**

The problem has three dimensions, where the Rubik solution is the issue of the input side. The base of water usage can be water, underground water, and purified wastewater. Let  $k = 1,2,3$  be the three players, who may follow variations of decision during their decision phase as follows:

The strategy for each player can be described by a five variable vector:

$$x_k = (f_k, t_k, k_k, f_k^*, t_k^*).$$

where

- $f_k$  = local water
- $t_k$  = local underground water
- $k_k$  = purified wastewater
- $f_k^*$  = import water
- $t_k^*$  = import underground water

The payoff-function for the total amount of water used for each player is as follows:

$$\phi_k = f_k + t_k + k_k + f_k^* + t_k^*$$

All players have two common constraining criteria, one of which state that the amount of used water may not be less than the minimal requirement  $D_k^{min}$ , while the other states that it may not be more than the maximum requirement of the technology either  $D_k$ . (These sustainability criteria are to avoid wasting water.)

$$f_k + t_k + k_k + f_k^* + t_k^* \geq D_k^{min}$$

$$f_k + t_k + k_k + f_k^* + t_k^* \leq D_k$$

In addition, the agricultural player ( $k=1$ ) has to introduce two additional criteria for water usage, which have the following variables:

$G$  = group of plants exclusive to underground water

$a_i$  = density of plants ( $i$ ) on the entire agricultural area

$w_i$  = water-demand of plants ( $i$ ) by acre

$T$  = group of plants which can be watered with purified wastewater

$W = \sum_i a_i w_i$  = total water – demand for all plants by acre

We know that the underground water supply offers the best quality water, while the purified wastewater offers the worst, so we have to define the volume of plants (sensitive) in the agricultural portfolio which can't be irrigated with purified wastewater. The water requirement which draws solely from the underground water sources may not exceed the water-dependence of the plants which are exclusive to clean, quality underground water:

$$\frac{t_1 + t_1^*}{f_1 + t_1 + k_1 + f_1^* + t_1^*} \geq \frac{\sum_{i \in G} a_i w_i}{W}$$

equation converted to linear form:

$$a_1 f_1 + (\alpha_1 - 1)t_1 + a_1 k_1 + \alpha_1 f_1^* + (\alpha_1 - 1)t_1^* \leq 0$$

$$\text{where } \alpha_1 = \frac{\sum_{i \in G} a_i w_i}{W}$$

Similarly, the rate and availability of the purified water can also be modeled. The water requirement for purified wastewater may not exceed the total available amount either. This connection gives the volume of plants that can either only or also be watered thus (f.e. plants for energetic use).

$$\frac{t_1}{f_1 + t_1 + k_1 + f_1^* + t_1^*} \geq \frac{\sum_{i \in T} a_i w_i}{W}$$

equation converted to linear form:

$$-\beta_1 f_1 - \beta_1 t_1 + (1 - \beta_1)k_1 - \beta_1 f_1^* - \beta_1 t_1^* \leq 0$$

$$\text{where } \beta_1 = \frac{\sum_{i \in T} a_i w_i}{W}$$

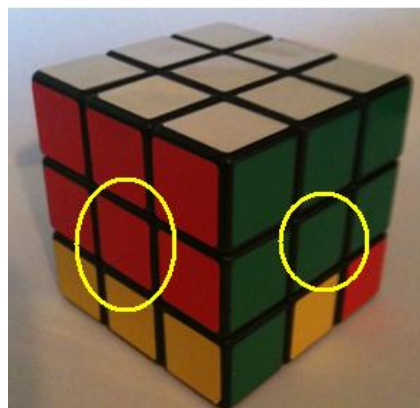
For the other players, we similarly have to define the connections of the functions defined by complications, for which the system can be found in the cited publications, before adding numeric data.

In light of the above mentioned facts, it can be stated that if we design our agricultural systems for the use of energetic biomass by allocating the complicating energy source (in this case, water) into an equilibrium state right at the beginning with Game Theory methods, then the planning process is applicable to the sustainability criteria system as well. The actual result of the entire analysis can be one of the following: either we won't over-calculate the water usage (over-calculate, as in the allocation won't be misrated), or we will discard the project entirely, because it doesn't abide by the sustainability criteria, since if it's clear at this point that the amount of water at hand is insufficient to reach the Pareto optimal production state, then the shortage of water causes a water-deficit in the analysed system.

### 3.4.2. Defining input and output connections with Game Theory correlations

#### GAME THEORY MODELING OF MID CUBE CONNECTIONS (LEVEL 2)

Keeping the mid cube in place and solving the row or layer imitates the zero sum game, since the position of the mid cube can not be changed, so it serves as a fix point for the rotation of the other cubes. Their place is fixed (meaning they can't be rotated out of their position, or connection systems), and their defined value elements can be considered constant (Illustration 3).



**Illustration 3: Zero sum games are always illustrated with the fixed mid cube (circled), which serve as criteria for the optimisation of outer cubes (two colors).**

Source: self-made.

#### Definition:

A  $J$  game with  $n$  – players is called a constant sum game, if the sum of the wins and losses of the player is a constant  $c$ , regardless of strategy.

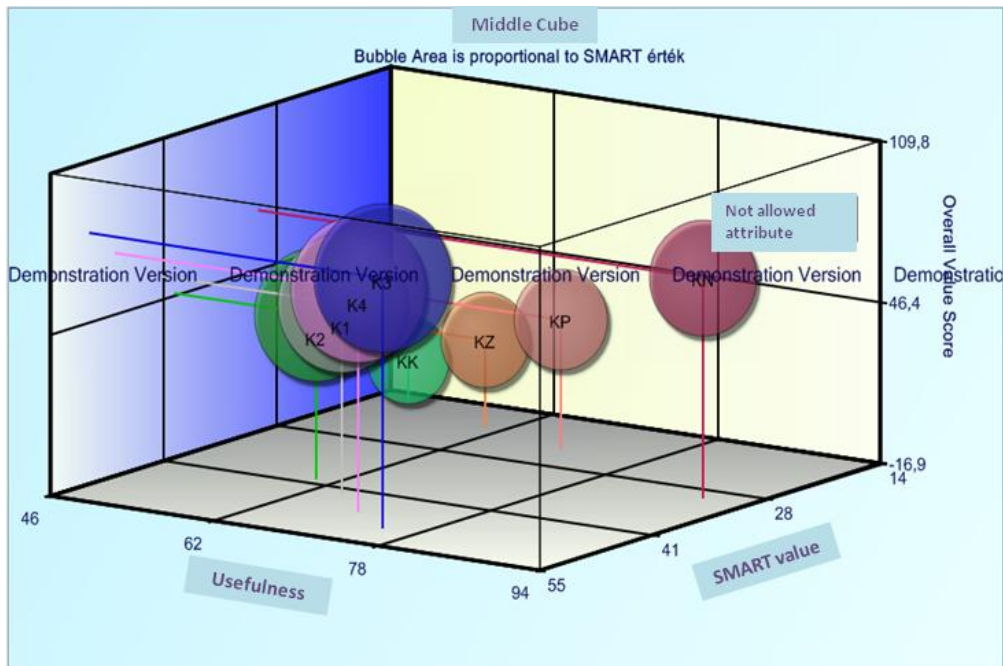
Formula:

$$\sum_{i=0}^n \varphi_i(x) = c \quad (x \in S).$$

Where  $c = 0$ , the game is *zero sum*.

**Thesis:**

With the zero sum game, we do a constant sum optimisation because the resource has a limited sum due to the fixpoint trait, therefore, the goal is to harmonically divide the resources at hand, and we search for the point of equilibrium of the attribute group (Illustration 4). During the SMART analysis, we verified that the orange mid cube of Rubik’s Cube shows a „not allowed difference” attribute. Currently, the inherent attribute group of the orange side is the monetary value of the project, and the time needed for payoff. The analysis of this trait with Game Theory optimisation methods shows us how the fixed resources of the low-carbon project will optimise themselves into a Nash equilibrium.



**Illustration 4: Not allowed attribute of the SMART analysis (time for payoff, value of project isn’t in equilibrium with the other attributes)**

**(Title: Middle Cube, Dimensions from left to right: Usefulness, SMART value, Value score).**

**Source: self-made**

The imbalance on Illustration 4 can be ascribable to the insufficiency of the stability of external factors which have an impact on the payoff of the investment. We have to analyse the circumstances of market entry of the newcomer.

It isn’t easy to solve the problem, if there are attributes in the group which are non-market elements (externals), but have an impact on the time required for payoff nevertheless (f.e. tax- and regulation policy, pollution control, foreign currency policy, etc.).

**Verification:**

I defined the points of Nash equilibrium for the mid cubes of Rubik’s Cube (four different fixed attributes) by searching for the attributes which aren’t part of the Pareto optimal state.

$n$  – player constant sum games can be used to demonstrate the points of equilibrium for the four different attributes.

If we take a  $(x_1^*, x_2^*, x_3^*, x_4^*) \in S$  point of equilibrium, we can define that

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^*, x_3^*, x_4^*) \text{ for every } x_1 \in S_1.$$

and

$$\varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_2(x_1^*, x_2, x_3^*, x_4^*) \text{ for every } x_2 \in S_2.$$

and

$$\varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_3(x_1^*, x_2^*, x_3, x_4^*) \text{ for every } x_3 \in S_3.$$

and

$$\varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_4(x_1^*, x_2^*, x_3^*, x_4) \text{ for every } x_4 \in S_4.$$

The game is zero sum, therefore

$$\varphi_1(x_1, x_2, x_3, x_4) + \varphi_2(x_1, x_2, x_3, x_4) + \varphi_3(x_1, x_2, x_3, x_4) + \varphi_4(x_1, x_2, x_3, x_4) = 0,$$

The second equality goes as follows

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) = 0$$

For either attribute to get a „not allowed difference” tag, as  $\varphi_1(x_1, x_2^* x_3^*, x_4^*)$

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^* x_3^*, x_4^*)$$

Prevalent for a constant sum game's every strategy as follows:

$$\begin{aligned} & \varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) \\ & \geq \varphi_1(x_1, x_2^* x_3^*, x_4^*) + \varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_3(x_1^*, x_2^*, x_3^*, x_4^*) + \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*) \end{aligned}$$

The point of equilibrium of the four player constant sum game ceases, if a shift in strategy happens for either of the factors:

$$(x_1^*, x_2^*, x_3^*, x_4^*) \rightarrow (x_1, x_2^*, x_3^*, x_4^*)$$

thus the shift in strategy (the change of any element of strategies) leads to inequality,

$$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^*, x_3^*, x_4^*)$$

This inequality-system states that if player one chooses a strategy different from  $x_i^*$ , and thus leaves the  $(x_1^*, x_2^*, x_3^*, x_4^*)$  equilibrium, and the game itself, his payoff-function can only be either equal to, or lower than that of the others. If the fourth player differs in a not allowed manner, but the others don't change their strategies, then his payoff-function will also be equal to, or lower compared to the  $\varphi_{1,2,3}(x_1^*, x_2^*, x_3^*, x_4^*)$  of the others.

$$\varphi_{1,2,3}(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_4(x_1^*, x_2^*, x_3^*, x_4^*)$$

Since this is a zero sum game, meaning the total payment can neither get higher or lower, the payoff-function of the  $\varphi_{1,2,3}(x_1^*, x_2^*, x_3^*, x_4^*)$  factors will either be equal to, or greater as well.



### 3.4.3. Imaging of Output-side algorithms

One of the popular types of non-cooperative Game Theory solutions is conflict alleviation methods. From these, we can highlight the axiomatic solution system of Nash, which creates axiom aggregations in order to assure the solution always places on the Pareto-line. The Kálmán-Smorodinsky solution defines the minimum reachable, or the last available point (meaning acceptable worst) to the solution of the conflict by defining the worst possible leaving point of the conflict.

#### GAME THEORY MODELING OF OUTPUT SIDE (LEVEL 3)

The phase of setting the final equilibrium state by the corner switch we do on the leaving side, the equilibrium search, and the finalisation of the sustainability criteria can usually only be done with cooperative strategy.

#### **Definition:**

*Cooperative games* can be defined by the following concepts.  $N = \{1, \dots, n\}$  as in *aggregation of players*, where  $S$  subset is known as a *coalition*:  $S \subseteq N$ . Let  $S$  be an aggregation of the subsets, meaning the aggregation of possible coalitions. The  $N$  main aggregation is called *coalition total*.

#### **Thesis:**

In low-carbon investment concepts, the project generates energy drawn from renewable sources, but the produced electricity can only reach the consumer if the owners of both the green electricity producer (Investor/B) and the electricity system (System/H) agree with each other that the product reaches the consumer through the system. A criteria of cooperation is that the investor pays a usage/transport fee to the owner of the system, and the owner acknowledges that instead of the previous (fossilized) product, he transports private product via the system, and in a lower volume. As compensation, the system gets the pay from the investor. This compromise, in essence, means that there has to be a valid agreement on provisioning conditions on the market. We tried to match the „green-yellow-orange” attribute cube of the previously established Rubik’s Cube project planning method with the model, and to assign the proper strategy to the cooperation.

#### **Verification:**

We can introduce our conflict-alleviation method with a two-player game. In the example, let the players’ strategies be represented by  $S_1$  and  $S_2$ , and the two payoff-functions by  $\varphi_1$  and  $\varphi_2$ . The aggregation of possible payoffs will therefore be  $2D$ , and can be shown as follows:

$$H = \{\varphi_1(x, y), \varphi_2(x, y) \mid (x, y) \in S_1 \times S_2\}$$

In this case, as always, the payoff of both players aims at maximalisation, but naturally, the various payoffs of one player depends on that of the other, and the fact that raising one player’s payoff will lower the other’s stands as a rule. Therefore, the objective is to find a solution, which is acceptable to both the investor and the system owner, meaning both parties simultaneously. We also have to state that in case of the agreement not being „signed”, both parties get a lower payoff, or a punishment.

Standard representations:

$$f_* = (f_{1*}, f_{2*})$$

this will be our standard payoff vector, where we assume that there is a  $(f_1, f_2) \in H$ , where  $f_1 > f_{1*}$ , and  $f_2 > f_{2*}$ . The problem is defined mathematically with the  $(H, f_*)$  pair. On Illustration

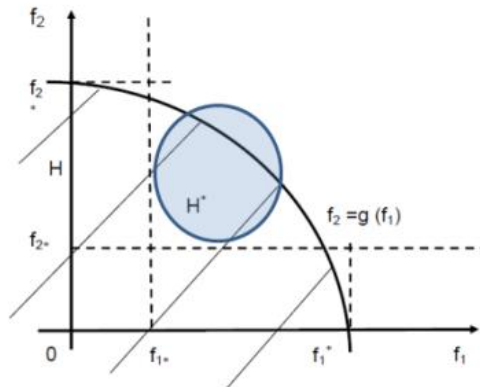
5, this pair was defined. We also assume that aggregation  $H$  is not open, convex, and bounded, so in case of:

$$(f_1, f_2) \in H \text{ and } \bar{f}_1 \leq f_1, \bar{f}_2 \leq f_2$$

$(\bar{f}_1, \bar{f}_2) \in H$  and bounded in both coordinates, meaning

$$\sup \{f_i | (f_1, f_2) \in H\} < \infty$$

in case of  $i = 1, 2$ .



**Illustration 5: Illustration of conflict state with the position of the payoff-function**

**Source: self-made**

We also assume that the limit of  $H$  is the graph of a  $f_2 = g(f_x)$  function, which is strictly falling in  $f_1$  and is concave. The graph of function  $g$  – is usually called the Pareto line, therefore, the conditions of satisfying the optimum criteria of sustainability can be met here. We must also take into consideration with the game and solution criteria that no rational player will accept a compromise that means a worse payoff than the payoff without agreement.

This way, we can tighten the payoff aggregation as follows:

$$H^* = \{f_1, f_2 | f_1 \geq f_{1*}, f_2 \geq f_{2*}, (f_1, f_2) \in H\}$$

*Conclusion:*

We concluded an unorthodox Game Theory optimum search on the different (cube) levels for the low-carbon planning of the project development process. During the Game Theory optimum search, I defined a theoretic model structure, which means fundamentally placing three different types of Game Theory solutions after each other, while keeping tabs on which Game Theory method is most efficient for featuring the various economic criteria systems:

1. Cube level one: non-cooperative three player game (for the correction of not allowed differences on Input side),
2. Cube level two: non-cooperative zero sum game (for the correction of not allowed differences of mid cube connections),
3. Cube level three: conflict alleviation method with two player game (for the correction of not allowed differences on Output side).

The three different Game Theory models together can define the states of Nash equilibrium required during project development, which help achieve sustainability during the realisation of the project. The sufficient selection of Nash equilibrium is possible through the SMART value definition based on the connection system of the cubes. An introduction to this will be given later in this document. However, we must stress that the Game Theory row that I selected (three person cooperative game, non-cooperative zero sum game, conflict-alleviation method) is applicable

mainly for typed energetic development, and a strictly defined economical environment (Hungary and East- and Mid-east Europe). Therefore, we can say that economic externals or development goals that differ from these can allow different Game Theory rows to be used as well.

### 3.5. SMART (Simple Multi Attribute Ranking Technic) analysis

To interpret the attributes of the cubes, and to define the attributes associable to the smallcubes, I chose the SMART (Simple Multi Attribute Ranking System) method, which can handle and illustrate 2D and 3D attributes at the same time. I chose the analysis method as defined in the methodology segment, which method counts as a one of a kind software application in terms of visually illustrating different attributes.

The process of the SMART analysis was as follows:

1. Evaluating the results of process of dominance conducted on main agents, input of data,
2. Defining the smallcube attributes of examination levels, and the estimated usefulness values,
3. Creation of SMART charts and illustration in 3D.

Using the results of the Churchman-Ackoff process of dominance analysis, the beginning data of the SMART evaluation is as follows (where I defined the color/attribute matches according to the results of said process of dominance analysis):

Group1: weighting of attributes		
Summarisation of monetary effects:	90 -----▶	orange
Adaptation of law and regulations:	70 -----▶	green
Examination of market opportunities:	60 -----▶	blue
<u>Technological criteria system:</u>	<u>50 -----▶</u>	<u>red</u>
Strategic program positioning:	30 -----▶	not present

In the first chapter, we answered the question, how we can match the solution algorithms with the different levels for the process of project planning based on Rubik's Cube, and with the Churchman-Ackoff method, we get the four most important attributes from the list of attributes which have an impact on it, namely those we can match to the cube's sides. If white (W) is the Input side, then the most dominant attribute group is matched with the orange (O) side, which gives us our WO base sidepair, from where we continue clockwise around the white side, and the following sidespairs of white-blue (WB), white-red (WR), and white-green (WG) will define the relevant attributes (agents) in the planning process.

The assortment criteria is that the most dominant attribute gets placed on the top (in our case, this is orange – O), and the least dominant attribute goes opposite to this side, meaning the bottom (in our case, this is red – R). The reason for this is that the description of their connection profiles (including the contradictions and errors) can be defined best, if it happens via a transaction through two other attributes (left and right sides). In our dominance list, the weakening attributes located in the middle are arranged by their „order of weakening”, namely counter-clockwise. The gist of the ranking is that the attribute groups that show stronger dominance are supposedly in better order, while the attribute group of weaker dominance is supposedly further from the point of equilibrium. The attribute group with the strongest relevance will be placed on the top, the weakest relevance attribute group will be opposite to this side, and finally, we define the remaining two groups by their „order of dominance”, namely, counter-clockwise. Since the solution of the cube usually happens clockwise in the various algorithms, the parts are optimised towards the point of equilibrium through the steps of process following the shortest route to solution, which explains why we position the most dominant attribute groups to the green side on the right of our beginning

orange side, and the third strongest dominant attribute group to the green side on the left of our beginning orange side.

*Definition of Usefulness-functions*

Compared to our analysis method, the SMART software offers a general function-definition method. The data and criteria introduced in the next structure can be simply added to the database of the program, and be evaluated with the help of the plugin algorithms.

SMALLCUBE NUMBER	USEFULNESS (1-100) (assumed)	CONNECTION, DIMENSION VALUE	SMART VALUE (V)	CUBE TYPE AND DOMINANCE	MAIN AGENT INHERENT ATTRIBUTES
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After filling in the charts, the conversion of usefulness-functions into usefulness values is conducted through the following steps:

1. the maximum value is assigned a score of 100,
2. the minimum value is assigned a score of 0,
3. after defining the two border values, we can also define the middle usefulness value (meaning half-useful compared to the maximum), which is assigned a score of 50,
4. after defining the maximum border value and the middle value, we can also define the usefulness value in-between them, which is assigned a score of 75,
5. after defining the minimum border value and the middle value, we can also define the usefulness value in-between them, which is assigned a score of 25.

Using a similar method, we can get inner function values, and to the points we get this way, we can mathematically assign an interpolation. The sufficiency of the alternatives is defined by the weighted mean of the usefulness value. I indicated the connection dimension values (which is also clearly visible in Chart 10) as follows: 3/3 for the three-level connections (cornercubes), 2/3 for three-level connections (outer cubes), and 1/3 for the fixed cubes. In regards to this, the usefulness-function for main graph or main attribute is as follows:

$$V(x_1, \dots, x_n) = \sum_i V_{i=\frac{1,2,3}{3^3}} \left( x_i = \frac{\sum_{i=0}^n w_i a_i}{\sum_{i=0}^n w_i} \right) \quad i = 1, \dots, n; \quad n = 9$$

$w_i = i - \text{number attribute's weight, } w_i > 0;$   
 $a_i = i - \text{attribute based value } a_i > 0$

According to the equation above, by portraying the SMART values for the various levels, we obtain clear knowledge on the attributes which have an impact on the different dimensions of usefulness for the main agent. We can view the inherent attributes of the main agents (O,R,G,B), their relations to each other, and the usefulness attributes of the Input side in Chart 10.

**Chart 10: Generating SMART input values for data insertion ('► X' = usefulness)**

MID WHITE (1D) – energy rationalisation	Dominance:100	(100) max value
OUTER WHITE (2D) Dominance: Orange/ 90; Green/70; Blue/60; Red/50		
strategy, regulation base connection (WG),	100/70	► 85,0
basic technological requirement (WR),	100/50	► 75,0
financing expectations (WO),	100/90	► 95,0
basic market positioning (WB)	100/60	► 80,0
CORNER WHITE (3D) –		
basic criteria of market payoff (WOB),	100/90/60	► 83,3
eligibility to monetary tools and regulation criteria (WOG),	100/90/70	► 86,6
syncing basic goals with technological threats and market priorities (WBR),	100/60/50	► 70,0
basic criteria designation of matching set of strategies and technologies (WGR),	100/70/50	► 73,3

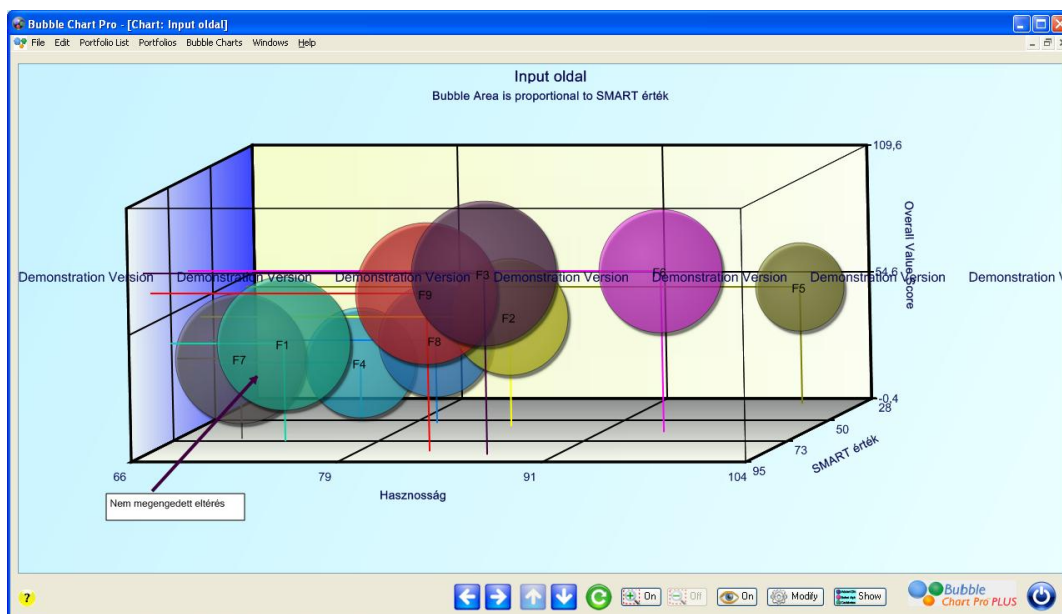
Source: self-researched

After the input of data generated in Chart 10, the SMART Bubble Chart Pro (demo version) creates the attribute illustrations via the „Value Score” point rating system, which is useful because the attributes compared to each other can be differentiated visually as well, regardless of that happening by their correctness or their strategic usefulness. Illustration 6 shows the input datachart of the SMART program.

Sel. No.	Name	Pic	Value Score	*A (100)	*B (100)	C (0)	D (0)	E (0)	F (0)	G (0)	H (0)	I (0)	J (0)	K (0)
6	F1		42,04	73	73	0	0	0	0	0	0	0	0	0
5	F2		47,22	85	57	0	0	0	0	0	0	0	0	0
1	F3		78,33	87	87	0	0	0	0	0	0	0	0	0
9	F4		24,07	75	50	0	0	0	0	0	0	0	0	0
4	F5		50	100	33	0	0	0	0	0	0	0	0	0
2	F6		69,44	95	63	0	0	0	0	0	0	0	0	0
8	F7		34,26	70	70	0	0	0	0	0	0	0	0	0
7	F8		35,19	80	53	0	0	0	0	0	0	0	0	0
3	F9		67,96	83	83	0	0	0	0	0	0	0	0	0
No. 9	Sum		449,5	748	569	0	0	0	0	0	0	0	0	0
	Median		47,2	83	63	0	0	0	0	0	0	0	0	0
	Mean		49,8	83	63	0	0	0	0	0	0	0	0	0
	Standard Deviation		18,4	10	17	0	0	0	0	0	0	0	0	0
	Maximum		78,3	100	87	0	0	0	0	0	0	0	0	0
	Minimum		24,1	70	33	0	0	0	0	0	0	0	0	0

**Illustration 6: 2D Illustration of the results chart of the SMART program**  
**Source: self-made based on SMART program**

The equilibrium state of the Input side is unstable, evidenced by the F1 attribute, which is the basic attribute designation of matching set of strategies and technologies (WGR/white-green-red). Illustration 7 shows the attributes, and their position in the attribute group. If we click on the sphere, we get the coordinates (x,y,z) for it, which translate to special usefulness-functions. Because of the 3D depiction, both the connection of attributes, and the depth of said connections can be easily interpreted on Illustration 7.



**Illustration 7: Depiction of results and non-equilibrium attributes of SMART program output**  
**Source: self-made**

With the aid of the SMART program, I evaluated the Input side (as seen on Illustrations 6 and 7), the mid cube side-attributes, and the Output side.

To summarise the analysis, we can say that I was successful in assigning the project development factors to the side colors of Rubik's Cube going by the dominance of the attribute groups. The process of dominance analysis conducted with the SMART pilot program, and the summary of its results is as follows:

- ✓ The definition of input and output sides of project attributes by picturing them to the white and yellow cube sides was completed,
- ✓ In case of the main attributes, orange was the most dominant, while red was the least dominant, therefore, the dominance values are the highest for orange, and the lowest for red. If the orange side is in the front, then we find the red side opposite to it, where green goes to the right, and blue goes to the left of orange,
- ✓ Going by the dominance, defined in the methodology part, the strongest attribute was assigned to orange, the second strongest assigned to green, the third strongest assigned to blue, while the weakest assigned to red, following the strength of dominance,
- ✓ I separated the analysis method (including the various attribute groups) into three different parts – 1) Input ► 2) Mid Cube ► 3) Output – for the sake of applicability of the Game Theory methods, and the SMART analysis,
- ✓ We can set the Game Theory optimisation for the „selection of technology for base criteria” attribute of the Input side, because this is where the SMART program showed a not allowed difference,
- ✓ We can set the Game Theory optimisation for the „monetary value of the project, and the time needed for payoff” attribute of the Mid cube side, because this is where the SMART program showed a not allowed difference,
- ✓ We can set the Game Theory optimisation for the „market criteria system, balancing of market instruments and provisions” attribute of the Output side, because this is where the SMART program showed a not allowed difference,
- ✓ Therefore, the results of the analyses on project evaluation of project planning processes for projects which aim at advancement from fossilized energy sources to renewable ones is that the attributes with the most impact are as follows: selection of technology for base criteria (Input side), which has the biggest impact on reaching equilibrium, monetary value of the project, and the time needed for payoff (Mid side, or connection attribute group) which if interpreted in a manner more suited to sustainability, will get us closer to the sustainable economical value, and market criteria system, balancing of market instruments and provisions (Output side), which needs proper and balanced planning, for the imbalances it causes may lead to a failed project.

### 3.6. NEW AND NOVEL SCIENTIFIC RESULTS

During the analysis of the sources, and the completed methodical analyses, I will introduce the results in two groups.  $M_n$  (New scientific conclusions) for the scientific results which can later be introduced as theses after further research or proper expansion of the analysed data, and  $T_n$  (New scientific results) for the ones that can be defined as scientific theses in the current research.

#### 3.6.1. New scientific conclusions

$M_1$ : The connection systems of project specifics which have an impact on the „shelf-life” and sustainability of enviro-orientated investments, or investments that have a positive impact on climate change can be defined through Rubik’s Cube’s „Layer by layer” solution method, and the Game Theory models assigned to the various phases of the method.

$M_2$ : The actual use of – including the designation of correct points of equilibrium for – Game Theory models for sustainable modeling of economical events often becomes harder, due to the many criteria which come into play. During the search for sustainable points of equilibrium with Game Theory models, the simple function-like definition of the connection systems of compared attributes, and the level-by-level handling of said connection systems based on simple planning phases makes realisation easier and more sufficient.

$M_3$ : I used multi-variable trial functions for the selection of attribute groups which can be defined as „having a negative impact on equilibrium of sustainable project planning and realisation”. In the various phases of project development – using process of dominance and usefulness-functions – helps correct this, and lead the project’s successful realisation towards the proper process.

$M_4$ : According to my hypotheses, low-carbon project development or planning processes can be called parallel with the layer by layer solution method of Rubik’s Cube. By assigning the various sides and colors to project attributes, we can realise a special and sustainable project development process, where the specifics of development and the process is achieved through the fact that the various attributes or attribute groups can be regarded as either one, two or three-dimensional system elements in the development programs. The same guiding theory is followed by low-carbon development and the solution system of Rubik’s Cube, which strives to reach the point of equilibrium through logic, and low energy input.

#### 3.6.2. New scientific results

$T_1$ : My comparison assessments verified that the various sustainability logics can be synchronised with the  $3 \times 3 \times 3$  Rubik’s Cube’s solution algorithms, where the relations of the cube’s sides define a planning strategy that provides a new scientific approach for investment planning. The first two solution algorithms out of the seven correlate with the Input factors of the investment planning process, the third and fourth describe the connection system of the starting and the finishing phases, while the remaining three can be associated with the attributes of the Output side.

$T_2$ : My analyses verified that the state of equilibrium for the *Input* side of the project development process, which also guarantees sustainability, can also be defined using a simple *constant sum game*, or *non-cooperative game of finite kind*. The connection systems, and the relative points of equilibrium of the *Input* and *Output* sides can also be described with *conflict alleviation method*, *zero sum game*, or *oligopolistic game of finite kind*. The point of equilibrium for the *Output* side of the development process can also be described with *cooperative oligopolistic game*, or cooperative strategies for *Nash equilibrium*.

T<sub>3</sub>: I verified with different calculations that the cornercubes of the 3×3×3 Rubik's Cube have a key role in the search process for point of equilibrium, or sustainability optimum. Their rotation combination, based on synchronising three attributes may offer investment programs a perfect Nash equilibrium. Without setting this, there's no final balance between the *Input* and *Output* sides, and the flexibility of the system is greatly lowered, since it didn't adapt the criteria which mean the ability of adapting to the possible changes in system attributes, and the relative elongation of „shelf-life”.

T<sub>4</sub>: One of the most valuable characteristics of the sustainability evaluation using the 3×3×3 Rubik's Cube is that it can handle the connection system and various usefulnesses assigned to the attributes defining the evaluation, simultaneously with all the other defining attributes (which are assigned to the remaining sides of the cube). It can define both the *two-dimensional* (x,y) and *three-dimensional* (x,y,z) connections, and can identify the attributes which have an impact on the various factors using the order of rotation. We can associate dual-colored outer cubes with 2D attributes, and tri-colored cornercubes with 3D interpretations, meaning it can also handle attributes which simultaneously belong to two, or three main agents.

T<sub>5</sub>: I also showed that the process of sustainable business planning can be defined with Game Theory models aptly, which model a sustainable project development process for the layer by layer solution method algorithms of Rubik's Cube. My examinations verified that a project development process can be considered sustainable, if the following criteria are met:

- A. The existence of a technologically sufficient planning option (to avoid over-planning and obsolescence)
- B. Optimisation of liquidity and financial sustainability is met (safe self-sufficiency and revenue for at least 10 years)
- C. Avoiding detrimental project effects on the relevant product areas (functionally self-sufficient system).



## 4. Conclusions and Suggestions

As it became obvious from the process of cited literature, we can find very different approaches on interpretations on sustainability in economics. It is therefore a basic dilemma when defining sustainability systems/strategies, if we should employ the theoretical connections of either *weak or strong sustainability* for the various interpretations. The difference between weak and strong sustainability is basically defined by the relations between environmental and artificial funds. The theory of weak sustainability states that environmental and artificial funds can replace each other. However, the relatively new index of total economic value (TEV) – which is an up-to-date economic adaptation which includes environmental value as well – still can't properly handle the resource transformation questions on the time-value of money. This question was sufficiently answered by the sustainable economic value (abbreviated as SEV), which is an estimation method that can – with the inclusion and goal-oriented use of local information – portray the changes in both environmental, social and technological fund-elements in an integrated manner, which is only partially realised by total economic value. Said total economic value (abbreviated as TEV) supports one of the newer economic branches, the so-called system of „low-carbon economy”.

The gist of the low-carbon economy concept is that it prefers structures which operate with low energy- and matter input on the level of the local economy or market, therefore viewing the criterir for long-term proper handling of resources as assured. Therefore, we can rightfully rely on the priority system of low carbon economy during the interpretation of criteria connected to sustainability, which is a concept aiming for equilibrium defined with what we can call the currently most advanced approach. Obviously, the sustainability interpretations search for solutions on the usage of resources, realisable concepts and actual models, the use of which will result in equilibrium. We may therefore think that to model sustainability in the future, the only sufficient scientific approaches are the ones that can offer well-defined and clear-cut solutions.

*Main conclusions of process of cited literature:*

- The economic interpretation of sustainability still has many inherent troubles, which take its imaging far from the level of operative realisation. Today, the sustainability criteria and economic criteria can most definetly be caught in the professional guides on low-carbon economy, which are considered acceptable enviro- or climate strategy guides by both political decision-makers and market participants.
- The question of handling and sustainable use of resources opened new possibilities on the front of economic equilibrium search in the last few years, one of which was the highly interesting and amusing solution of using Game Theory methods. Despite the many disappointments and losses of trust Game Theory strategy models suffered in terms of economical decisions, we can say that the new theoretical approaches, new ways on finding the proper Nash equilibrium, and the use of simplified models is a reassuring occurence coming from the field of mathematic modeling.
- During the process of cited literature, I already mentioned the new low-carbon software development approach, which can mean a new recycling method for old and out-of-date software. We can satisfy new consumer needs with our software applications with this solution, while using much less input of materials and effort, which means re-entering the market with our „dead” product. The secret of this software-regeneration is using the layer by layer, meaning row by row solution method of Rubik's Cube as a base, which imitates the solution of the 3×3×3 Rubik's Cube, thereby „knocking” the development process into the proper state of equilibrium through a multitude of short interactions, which also correlates with the new consumer needs.

### *Main conclusions of my personal research:*

- The sustainability criteria and the classic „layer by layer” solution method of Rubik’s Cube can be synchronised with each other. The analysis of connections between the basic build, the sustainability interpretation, and the solution algorithms of Rubik’s Cube verify that the process leading to the cube’s configurational balance is a process of search for equilibrium, which in our case is also applicable to model the equilibrium search of enviro-defense processes.
- My analyses on Game Theory strategy models show that in today’s practice, we can find a multitude of economic strategy models that don’t really work as intended. The reason for this is basically the over-complication of the models, and the inclusion of the many-many factors, and criteria. In order to save the process of modeling and the actual mechanisms of the models from falling into the category of „too complicated, no thanks”, we require a simplified, and yet correctly working model that is easily interpretable, properly loadable with different data, and easily correctable. My three-level, usable with Game Theory models „unorthodox Game Theory optimum search” model offers a solution for this challenge.
- During the analysis of the Game Theory models, and the strategic optimum search systems, I made the conclusion that it’s more beneficial to use smaller, individual and unique Game Theory solutions which have different reactions in a business environment to describe the process of equilibrium search, instead of using complex multi-factor model-structures to functionate the entire process. In case of development processes for renewable energy production, in other words, advancement from fossilized to renewable energy sources, by dividing the development program to three levels, then using *non-cooperative Game Theory method* for the first, *constant or zero sum game* for the second, and finally, to define output criteria, *cooperative Nash-equilibrium search with multi-player oligopolistic game* for the third offers a more beneficial result.

The unorthodox Game Theory method I described suggests – during the phase of actual use – that we use function characteristics which are flexible time-wise for the various levels (input, mid cube/connection, and output), therefore, it may prove applicable to model more complicated processes, if we form an optimum search process by a consecutive use of many simple Game Theory models. These methods/games can also be changed, and flexibly adapted to different economic criteria systems, according to the changes in business environment.

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